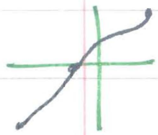
## 4.2 The Mean Value thm.

**Rolle's thm** If $f(x)$ is continuous on a closed interval $[a,b]$ and differentiable on the open interval $(a,b)$ and if $f(a) = f(b)$ then $f'(c) = 0$ for at least one number $c \in (a,b)$.

**Ex.** Show that $x^3 + 3x + 1 = 0$ has exactly one real solution.

$f(x) = x^3 + 3x + 1$ is polynomial so it is cont & diff $\forall$ $\mathbb{R}$.



$f(-1) = -3$, $f(0) = 1$ $\Rightarrow$ from intermediate value thm. $\exists\, c \in (-1, 0)$ s.t. $f(c) = 0$

$$f'(x) = 3x^2 + 3 > 0 \quad \forall x \in \mathbb{R}$$

always increasing

also from Rolles thm. Since $f'(c) \neq 0$ $\forall c \in \mathbb{R}$

$\Rightarrow \nexists\, a, b \in \mathbb{R}$ such that $f(a) = f(b)$

$\Rightarrow f(x)$ has only one zero.

**Ex.** $f(x) = x^4 - 3x^2 + 2x + 1$ for $[0,1]$

Show that $c \in (0,1)$ s.t $f'(c) = 0$

$f(x)$ poly. $\Rightarrow$ cont & diff on $[0,1]$

from Rolle's since $f(0) = f(1) = 1$ $\exists\, c \in (0,1)$ $\Rightarrow f'(c) = 0$.

# The Mean Value th°

$f(x)$ cont $[a,b]$ & diff $(a,b)$, then $\exists\ c \in (a,b)$

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Ex. Show that $f(x) = x^3 - 8 + 2x$ satisfies the MVT. then find c.

$f(-4) = f(2) = 0$

$f'(x) = 2x + 2$

$f'(c) = 0$

$2c + 2 = 0 \qquad \underline{c = -1}$

Ex. (Rolle's) $\quad f(x) = \sin 2x \quad [\pi/4, \frac{5\pi}{4}]$

$f(\pi/4) = 1 \qquad f(\frac{5\pi}{4}) = 1$

$f'(c) = 0 \qquad 2\cos 2c = 0 \qquad c = \frac{3\pi}{4}$

Th° If $f'(x) = 0$, then $f$ is constant on $(a,b)$

i.e $f(x) = k$.

Th° If $f'(x) = g'(x)$ then

$f(x) = g(x) + k$.

Ex. find $f(x)$ whose derivative $3x^2$ passes through $(0,0)$

$f(x) = x^3 + c \qquad f(0) = x^3 + c = 0$

$\Rightarrow c = 0$

So $f(x) = x^3$