4.1 Maximum and minimum Values

Def: $f(x)$ where $x \in D \subset$ Domain. $c \in D$.

A. $f(c)$ is Absolute maximum value of f if $f(c) \geq f(x)$

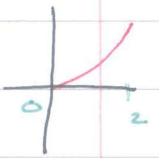
B. $f(c)$ is $\leq$ minimum value of f if $\forall\, x \in D$

$f(c) \leq f(x) \quad \forall x \in D$

Ex. $f(x) = x^2 \quad [0, 2]$

Abs min at $x = 0$

Abs max at $x = 2$.



Def

A. $f(c)$ is local max of f if $f(c) \geq f(x)$ when x near c

B. $f(c)$ is local min of f if $f(c) \leq f(x)$ " " " "

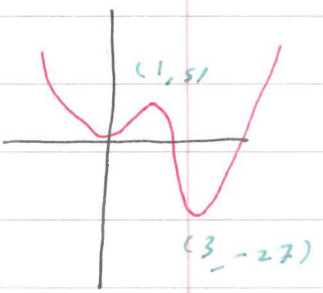
Ex. $f(x) = \sin x$

$x = \frac{\pi}{2} + 2n\pi$ all Abs max

$x = \frac{3\pi}{2} + 2n\pi$ all Abs min

Ex.

$$f(x) = 3x^4 - 16x^3 + 18x^2$$



$x = 3$ is Abs min No Abs max

$x = 1$ is local max no local min

Th (The extreme value th')

If $f(x)$ is cont. on $[a, b]$, then $f(x)$ has both Abs min & Abs max.

Th If $f(x)$ has local extrema at c and if $f'(c)$ exist, then $f'(c) = 0$.

Df $c \in D$ is a Critical point (CP) of $f(x)$ if $f'(c) = 0$ or $f'(c)$ DNE

How to find Abs extrema

① Evalute C.P. & end points

② the largest is Abs max & the Smallest is Abs min.

Ex. find C.P. of $f(x) = x^{3/5}(4-x)$

Domain $\mathbb{R}$ (important) Not in the Domain not C.P.

$$f'(x) = \frac{12-8x}{5x^{2/5}}$$

$f'(c) = 0 \Rightarrow x = 3/2$

$f'(c)$ DNE $x = 0$ (it is in the Domain)

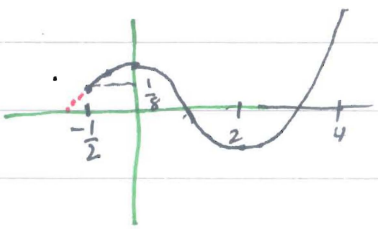
Ex. Find Abs extreme

(a) $f(x) = x^3 - 3x^2 + 1 \qquad [-1/2, 4]$

$f'(x) = 3x^2 - 6x = 3x(x-2)$

C.P $x = 0, \; x = 2$

x	$f(x)$	
$-1/2$	$1/8$	
0	1	
2	-3	Abs min
4	17	Abs max



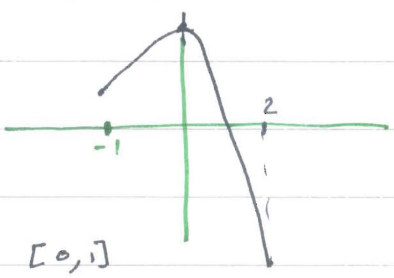
(b) $f(x) = 5 - 6x^2 - 2x^3 \qquad [-1, 2]$

$f'(x) = -12x - 6x^2 = -6x(2+x)$

C.P $x = 0, \; x = -2$

Now $x = -2$ not in the domain

x	$f(x)$	
-1	1	Abs min
0	5	
2	-35	Abs max



(c) $f(x) = 1 - x^{2/3} \qquad [0, 1]$

$f'(x) = -\frac{2}{3}x^{-1/3} = \frac{-2}{3\sqrt[3]{x}}$ C.P $x = 0$

A max
$x = 0 \quad f(x) = 1$
$x = 1 \quad f(x) = 0$
A min

(d) $f(x) = \sin^2 x - \cos x$

$f'(x) = 2\sin x \cos x + \sin x$

$= \sin x(2\cos x + 1)$

if $[0, \pi]$
$0, \pi$
$2\pi/3$

$\sin x = 0$ if $x = n\pi$

$2\cos x + 1 = 0$ if $\cos x = -\frac{1}{2}$ if $x = \frac{2\pi}{3} + 2n\pi$
$4\pi/3 + 2n\pi$