

Name: -

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Q1. Show that  $\tanh(\ln x) = \frac{x - x^{-1}}{x + x^{-1}}$

$$\tanh(\ln x) = \frac{e^{\ln x} - e^{-\ln x}}{e^{\ln x} + e^{-\ln x}} = \frac{x - x^{-1}}{x + x^{-1}}$$

Q2. Find all the values of  $c$  in  $(-2, 1)$  that satisfy the conclusion of the Mean Value Theorem for  $f(x) = x^2$

$$f'(x) = 2x$$

$$f'(c) = 2c = \frac{f(1) - f(-2)}{1 - (-2)} = -1$$

$$c = -\frac{1}{2}$$

Q3. Find  $\lim_{x \rightarrow \infty} x^{\frac{1}{x}}$

$$y = \lim_{x \rightarrow \infty} x^{\frac{1}{x}}$$

$$\ln y = \lim_{x \rightarrow \infty} \frac{1}{x} \ln x \left( \frac{\infty}{\infty} \right)$$

$$= \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{1} = 0$$

$$y = e^0 = 1$$