Quiz 8(Class Test) (Dr. AL-Homidan)(171) MATH 101-10

Name:- ID:- Key①

Q1.

If $f(x)=\dfrac{ax+b}{cx+d}$, where a,b,c and d are constants, then $f'(x)=$

a. $\dfrac{2acx+(ad+bc)}{(cx+d)^2}$

b. $\dfrac{a+b}{cx+d}$

c. $\dfrac{ad-bc}{(cx+d)^2}$

d. $\dfrac{acx+(ad-bc)}{(cx+d)^2}$

e. $\dfrac{(a+c)x-(b+d)}{(cx+d)^2}$

$$= \frac{a(cx+d)-c(ax+b)}{(cx+d)^2} = \frac{a\cancel{cx}+ad-\cancel{cax}-cb}{(cx+d)^2} = \frac{ad-cb}{(cx+d)^2}.$$

4^{x^2}

Q2.

If $f(x)=\cos^{-1}(4^{x^2-3}-4)$, then $f'(2)=$

a. 8

b. $-8\ln 4$

c. $-16\ln 4$

d. $\dfrac{-8\ln 4}{\sqrt{2}}$

e. $\dfrac{\ln 4}{\sqrt{2}}$

$$f'(x)=\frac{-(4^{x^2-3}\,2x\ln 4)}{\sqrt{1-(4^{x^2-3}-4)^2}}$$

$$f'(2)=\frac{-(4^{4-3}\,4\ln 4)}{\sqrt{1-(4-4)^2}} = -8\ln 4$$

Q3.

Key ②

Suppose that L is a function such that $L'(x) = \frac{1}{x}$ for $x > 0$.

Then the derivative of $F(x) = L(x^4) + L\left(\frac{1}{x}\right)$ is equal to

$F'(x) = \frac{1}{x^4} 4x^3 + \frac{1}{\frac{1}{x}} -x^{-2}$

(a) $x^4 - x$

(b) $\frac{5}{x}$

$= \frac{4x^3}{x^4} - \frac{x}{x^2} = \frac{4}{x} - \frac{1}{x} = \frac{3}{x}$

(c) x^3

(d) $\frac{3}{x}$

(e) $\frac{4}{x^3}$

Q4.

$\ln y = x \cos x \ln x$

An equation of the normal line to the graph of $y = x^{x \cos x}$ $\Rightarrow$ when $x = \frac{\pi}{2}$ is given by

$y(\frac{\pi}{2}) = \frac{\pi}{2}^{\frac{\pi}{2}\cos\frac{\pi}{2}} = \frac{\pi}{2}^{(0)} = 1$

(a) $2\pi(\ln\sqrt{\pi} - \ln\sqrt{2})(y-1) = 2x - \pi$

$\frac{y'}{y} = \cos x \ln x + x(-\sin x)\ln x + \frac{x\cos x}{x}$

(b) $\pi \ln\sqrt{\pi}(y-1) = (\ln 2)x - \pi$

(c) $(\ln\sqrt{\pi} - \ln\sqrt{2})(y-1) = 2x - \pi$

$y'(\frac{\pi}{2}) = y(\frac{\pi}{2})[0 + (-\frac{\pi}{2})\ln\frac{\pi}{2} + 0]$

(d) $\pi(\ln\sqrt{\pi} - \ln\sqrt{2})(y-\pi) = x - 1$

$y'(\frac{\pi}{2}) = -\frac{\pi}{2}\ln\frac{\pi}{2} = m_{slope}$

(e) $2\pi\ln(\pi - 2)(y-1) = x - \pi$

$m_{norm} = \frac{1}{\frac{\pi}{2}\ln\frac{\pi}{2}}$

$\frac{\pi}{2}\ln\frac{\pi}{2}$
$= \pi\ln(\frac{\pi}{2})^{\frac{1}{2}}$
$= \pi[\ln\sqrt{\pi} - \ln\sqrt{2}]$

$y = m(x - x_0) + y_0$

$y = \frac{1}{\frac{\pi}{2}\ln\frac{\pi}{2}}(x - \frac{\pi}{2}) + 1 \Rightarrow (y-1) = \frac{1}{\frac{\pi}{2}\ln\frac{\pi}{2}}(x - \frac{\pi}{2})$

$\Rightarrow 2\pi[\ln\sqrt{\pi} - \ln\sqrt{2}](y-1) = 2x - \pi$

Q5.

Key ③

If the position of a particle is given by the equation $s(t) = -\frac{1}{3}t^3 + \frac{3}{2}t^2 - 2t + 1,\ 0 \le t \le 5,$ then the particle moves in **the negative direction** during the time interval(s) [t is measured in seconds and s in meters]

$t^2 - 3t + 2$

a. (0, 1) and (2, 5)
b. (1, 2) only
c. (0, 1) and (1, 2)
d. (2, 5) only
e. (1, 2) and (2, 5)

$v(t) = -t^2 + 3t - 2 = 0$

$v(t) = (t-1)(t-2)$

0 − 1 + 2 −

$(0,1), (2,5)$

Q6.

The number of points on the curve $y = \frac{1}{x^4 + x^2 + 1}$ at which the tangent line is horizontal is

a. Zero
b. Two
c. Three
d. Four
e. One

$y' = -(x^4 + x^2 + 1)^{-2}(4x^3 + 2x)$

$y' = \frac{-(4x^3 + 2x)}{(x^4 + x^2 + 1)^2} = 0$

$2x(2x^2 + 1) = 0$

$x = 0 \qquad x = \pm \frac{1}{\sqrt{2}}$

Q7.

Key (4)

If the polynomial $P(x) = ax^3 + bx^2 + cx + d$ satisfies the conditions

$P(1) = 1, P'(1) = 3,\ P''(1) = 6$ and $P'''(1) = 12$, then $abcd =$

a. -18

b. 12

c. 18

d. 36

e. -9

$P'(x) = 3ax^2 + 2bx + c$

$P''(x) = 6ax + 2b$

$P'''(x) = 6a \qquad P'''(1) = 6a = 12$

$\boxed{a = 2}$

$P''(1) = 12 + 2b = 6$

$2b = -6 \qquad \boxed{b = -3}$

$P'(1) = 6 + -6 + \boxed{c = 3}$

$d = -1$

$P(1) = 2 - 3 + 3 + d = 1$

Q8.

.The volume of a sphere is increasing at a rate of $6\ \text{cm}^3/\text{sec}$. The rate of change of its surface area when its volume is $\frac{256\pi}{3}$ cm^3 is [Hint: $V = \frac{4\pi}{3}r^3$ and $S = 4\pi r^2$]

a. 3

b. $\frac{3}{8}$

c. 2

d. $\frac{64}{3}$

e. $\frac{3}{4}$

$V = \frac{4\pi}{3}r^3 \qquad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 6 \qquad \frac{dS}{dt}$? when $V = \frac{256\pi}{3}$

$\frac{256\pi}{3} = \frac{4\pi}{3}r^3$

$\frac{4(64)\pi}{3} = \frac{4\pi}{3}r^3$

$r^3 = 64$

$r = 4$

$6 = 4\pi\, 16 \frac{dr}{dt}$

So $\frac{dr}{dt} = \frac{6}{4\pi 16}$

So we need $\frac{dS}{dt}$ when $r = 4$

$\frac{dS}{dt} = 4\pi\, 2r \frac{dr}{dt}$

then $\frac{dS}{dt} = 4\pi\, 2(4) \frac{6}{4\pi 16} = \frac{6}{\cancel{2}} = 3$

Q9.

Key (5)

$f(t)=\dfrac{\tan t}{1+\sec t}$, then $f'(t)=$

a. $\dfrac{\sec^2 t}{1+\sec t}$

b. $\dfrac{\sec t}{(1+\sec t)^2}$

c. $\dfrac{\sec t}{1+\sec t}$

d. $\dfrac{\sec t+\tan t}{(1+\sec t)^2}$

e. $\dfrac{\sec t\,\tan^2 t}{(1+\sec t)^2}$

$$f'(t)=\frac{\sec^2 t\,(1+\sec t)-\tan t\,\sec t\,\tan t}{(1+\sec t)^2}$$

$$=\frac{\sec^2 t+\sec^3 t-\tan^2 t\,\sec t}{(1+\sec t)^2}$$

$$=\frac{\sec^2 t+\sec^3 t-(\sec^2 t-1)\sec t}{(1+\sec t)^2}$$

$$=\frac{\sec^2 t+\cancel{\sec^3 t}-\cancel{\sec^3 t}+\sec t}{(1+\sec t)^2}$$

$$=\frac{\sec t\,\cancel{[\sec t+1]}}{(1+\sec t)^{\cancel{2}}}$$

Q10.

$y=\ln\sqrt{\dfrac{1+\sin x}{1-\sin x}}$, then $\dfrac{dy}{dx}=$

a. $\tan x$

b. $\sec x$

c. $\cot x$

d. $\sin x$

e. $\cos x$

$$y=\frac{1}{2}\ln(1+\sin x)-\ln(1-\sin x)$$

$$y'=\frac{1}{2}\left[\frac{\cos x}{1+\sin x}-\frac{-\cos x}{1-\sin x}\right]$$

$$y'=\frac{1}{2}\left[\frac{\cos x(1-\sin x)+\cos x(1+\sin x)}{1-\sin^2 x}\right]$$

$$y'=\frac{1}{2}\left[\frac{\cos x-\cancel{\cos x\sin x}+\cos x+\cancel{\cos x\sin x}}{\cos^2 x}\right]$$

$$=\frac{1}{\cancel{2}}\left[\frac{\cancel{2}\cancel{\cos x}}{\cos^2 x}\right]=\frac{1}{\cos x}$$

Q11.

Key 6

There are two lines through the point $(2, -3)$ that are tangent to the parabola $y = x^2 + x$. Then the **sum** of the slopes of these lines is

let x_0 is the coordinate ~~of~~ of tangent point

(a) 11

then

(b) 13.5

$$m = \frac{y_0 + 3}{x_0 - 2} \quad \& \quad y'(x_0) = m = 2x_0 + 1$$

(c) 7.5

$$\Rightarrow \quad 2x_0 + 1 = \frac{y_0 + 3}{x_0 - 2}$$

but x_0 satisfies $y = x^2 + x$

$$y_0 = x_0^2 + x_0$$

(d) 10

(e) 9

$$\Rightarrow (2x_0 + 1)(x_0 - 2) = x_0^2 + x_0 + 3$$

$$2x_0^2 - 3x_0 - 2 = x_0^2 + x_0 + 3 \qquad x_0^2 - 4x_0 - 5 = 0$$

$$(x_0 + 1)(x_0 - 5)$$

$x_0 = -1 \quad m_1 = 2(-1) + 1 = -1$

$x_0 = 5 \quad m_2 = 2(5) + 1 = 11$

$+ \; = 10$

Q12.

If $\sqrt{x} + \sqrt{y} = 4$ defines implicitly a relation between x and y, then y'' is equal to

$$\frac{1}{2\sqrt{x}} + \frac{y'}{2\sqrt{y}} = 0$$

$$y' = -\sqrt{y/x}$$

(a) $\dfrac{\sqrt{xy}}{2x^2y}(x+y)$

$$y'' = -\frac{1}{2}\sqrt{\frac{x}{y}} \cdot \frac{y'x - y}{x^2}$$

(b) $\dfrac{xy + y\sqrt{xy}}{2x^2y}$

$$y'' = -\frac{1}{2}\sqrt{\frac{x}{y}} \quad \frac{-\sqrt{\frac{y}{x}}\,x - y}{x^2}$$

(c) $-\sqrt{\dfrac{y}{x}}$

(d) $-\sqrt{\dfrac{x}{y}}$

$$y'' = +\frac{1}{2}\sqrt{\frac{x}{y}} \cdot \frac{(\sqrt{yx} + y)}{x^2} \cdot \frac{\sqrt{y}}{\sqrt{y}}$$

(e) $\dfrac{x\sqrt{y} + y\sqrt{x}}{2x^2}$

$$= \frac{1}{2}\,\frac{xy + y\sqrt{xy}}{y\,x^2}$$

Q13.

Key 7

The value of the limit $\lim_{\theta \to 0} \frac{\sin(2\theta)}{\theta + \tan(4\theta)}$ is equal to

a. $\frac{1}{2}$

b. $\frac{2}{3}$

c. $\frac{5}{6}$

d. $\frac{2}{5}$

e. $\frac{3}{4}$

$$\lim_{\theta \to 0} \frac{\sin 2\theta}{\frac{\theta \cos 4\theta + 2\sin 2\theta \cos 2\theta}{\cos 4\theta}}$$

$$\Rightarrow \lim_{\theta \to 0} \frac{\frac{\sin 2\theta}{2\theta} \cos 4\theta}{\frac{\theta \cos 4\theta + 2 \sin 2\theta \cos 2\theta}{2\theta}} = \cancel{\frac{1 \cdot 1}{\frac{1+2}{4}}}$$

$$= \frac{1 \cdot 1}{\frac{1}{2} + 2} = \frac{1}{\frac{5}{2}} = 2/5$$

Q14.

.If $f(x) = \frac{1}{3-4x}$, then $f^{(2008)}(1) =$

a. $(-1) \cdot 4^{2008} \cdot (2008)!$

b. $(2008)!$

c. $4^{2008} \cdot (2008)!$

d. $\frac{(-1) \cdot (2008)!}{4^{2008}}$

e. $(-1) \cdot (2009)!$

$$f(x) = (3-4x)^{-1}$$

$$f'(x) = -(3-4x)^{-2}(-4)$$

$$f''(x) = (-1)(-2)(3-4x)^{-3}(-4)(-4)$$

$$f^{(n)} = (1 \cdot 2 \cdot 3 \ldots n)(3-4x)^{-n-1} 4^n$$

$$f^{(2008)}(1) = 2008! \, (-1)^{2009} \, 4^{2008}$$

$$= 2008! \, (-1) \, 4^{2008}$$