Quiz 6 (Dr. AL-Homidan)(171) MATH 101-10

Name:- ID:-

Q1. Given $\sqrt{x}+\sqrt{y}=1$,

i. Find equation of the tangent line at $x=1$. $\Rightarrow \sqrt{1}+\sqrt{y}=1$
$y=0$

Hint Point =(1,?) $(1,0)$

Slope= $y'(1,?)$=

y'

$$\frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}y'=0 \Rightarrow \frac{1}{2\sqrt{y}}y'=-\frac{1}{2\sqrt{x}}$$

$$\Rightarrow y'=-\sqrt{y/x}$$

$$\text{so } y'\Big|_{(1,0)}=-\sqrt{0/1}=0$$

Eq. $y=0(x-1)+0 \Rightarrow \boxed{y=0}$

ii. Find y''.

$$y''=-\frac{1}{2\sqrt{y/x}}\cdot\left(\frac{y'x-y}{x^2}\right)=\frac{-\frac{1}{2}\sqrt{\frac{x}{y}}\,y'x+\frac{1}{2}\sqrt{\frac{x}{y}}\cdot y}{x^2}$$

$$y''=\frac{-\frac{1}{2}\sqrt{\frac{x}{y}}\left(-\sqrt{\frac{y}{x}}\right)x+\frac{1}{2}\sqrt{xy}}{x^2}=\frac{x}{2x^2}+\frac{\sqrt{xy}}{2x^2}$$

$$y''=\frac{\sqrt{x}}{2x^2}\left[\sqrt{x}+\sqrt{y}\right]$$

$$=\frac{1}{2x\sqrt{x}}\left[\sqrt{x}+\sqrt{y}\right]$$

Q2. Find y' if $\tan^{-1}(xy)=x-e^{y}$.

$$\frac{1}{1+(xy)^2}\cdot(y+xy')=1-e^{y}y'$$

$$\Rightarrow \frac{y}{1+(xy)^2}+y'\frac{x}{1+(xy)^2}=1-e^{y}y'$$

$$y'\left[\frac{x}{1+(xy)^2}+e^{y}\right]=1-\frac{y}{1+(xy)^2}$$

$$y'=\frac{1+(xy)^2-y}{x+e^{y}+e^{y}(xy)^2}$$