MATH-101-10 term 171 Quiz 4 Key

Name:- ID:-

Q1. *If* $h(x)=e^{2x}$ and n is a positive integer, then find $f^{(n)}(1)$

$h'(x)=2e^{2x}$ $\quad h''(x)=2.2\,e^{2x}$ $\quad h^{(n)}=\underbrace{2\ldots2}_{n}\,e^{2x}$

$h^{(n)}(x)=2^n e^{2x}$

$h^{(n)}(1)=2^n e^2$

Q2. Let f and g be differentiable functions and $h(x)=f(g(k(2x)))$ and if $k(4)=1$, $k'(4)=2$, $g(1)=3$, $g'(1)=0.5$ and $f'(3)=0.5$ then find $h'(2)$

$h'(x)=f'(g(k(2x)))\cdot g'(k(2x))\,k'(2x)\cdot 2$

$=1$

Q3. Find $\lim\limits_{x\to 0}\left(\dfrac{x+\tan x}{\sin x}\right)$

$\lim\limits_{x\to0}\dfrac{x}{\sin x}+\lim\limits_{x\to0}\dfrac{\sin x}{\sin x}\cdot\dfrac{1}{\cos x}$

$=\lim\limits_{x\to0}\dfrac{1}{\frac{\sin x}{x}}+\lim\limits_{x\to0}\dfrac{1}{\cos x}=\dfrac{1}{1}+1=2$

Q4. The equation of the normal line to the graph of the curve $y=\dfrac{1+\sin x}{\cos x}$ at $(\pi,-1)$ is

$y'=\dfrac{\cos x\cos x-(1+\sin x)(-\sin x)}{\cos^2 x}$

$=\dfrac{\cos^2x+\sin^2x+\sin x}{\cos^2x}=\dfrac{\sin x+1}{\cos^2x}$

$y'(\pi)=\dfrac{\sin\pi+1}{\cos^2\pi}=\dfrac{0+1}{(-1)^2}=1$ ← Slope of the tangent line

$m_n\cdot m_t=-1 \Rightarrow$ Slope of the normal line $m_n=-1$

Equation of normal line $y=-(x-\pi)-1=-x+\pi-1$