Quiz 3

Solution

Name:- ID:-

Q1.

The value(s) of constant c making the function

$$g(x) = \begin{cases} x^2 - c^2 & \text{if } x < 4 \\ cx + 20 & \text{if } x \geq 4 \end{cases}$$

continuous on $(-\infty, \infty)$ is (are)

(a) 4 and -4

(b) -2

(c) 20

(d) -2 and 2

(e) 4

$g(4) = 4c + 20$

must equal to $\lim_{x \to 4^-} g(x)$

$= 16 - c^2$

$4c + 20 = 16 - c^2$

$c^2 + 4c + 4 = 0$

$c = -2$

Q2.

If the line $2x + y = b$ is tangent to the parabola $y = ax^2$ at $x = 2$, then $a - b =$

(a) $-\frac{3}{2}$

(b) $-\frac{5}{2}$

(c) $\frac{1}{2}$

(d) $\frac{3}{2}$

(e) $-\frac{1}{2}$

$\frac{dy}{dx} = 2ax$ $m = 4a$

$\|$

$2x + y = b$ $m = -2$

$a = -\frac{1}{2}$

but $x = 2$ $y = 4a$

then

$2(2) + 4a = b$

$4 + (-2) = b$

$b = 2$

$a - b = -\frac{1}{2} - 2$

Q3.

$2X-3<0$ when $X\to 1.5^-$

$\lim\limits_{x\to 1.5^-} \dfrac{2x^2-3x}{|2x-3|}$ is equal to

$\lim\limits_{X\to 1.5^-} \dfrac{X(2X-3)}{|2X-3|} = \lim\limits_{X\to 1.5} -x = -1.5$

(a) 3

(b) 1

(c) $\frac{3}{2}$

(d) -1

(e) $-\frac{3}{2}$

Q4.

To prove that $\lim\limits_{x\to 2}(2x-1)=3$ by using the $\varepsilon-\delta$ definition of the limit, we find that for given $\varepsilon=0.002$, the largest possible value for δ that can be used is

(a) 0.001

$|2X-1-3|<0.002$

(b) 0.05

$|2X-4|<0.002$

(c) 0.002

$2|x-2|<0.002$

(d) 0.003

(e) 0.02

$\delta = 0.001$

Q5.

The sum of all values of k, for which $y = k$ is a horizontal asymptote to the graph of the function

$$f(x) = \begin{cases} \dfrac{2+\sqrt{x}}{2-\sqrt{x}} & \text{for} \quad x > 4 \\ 1 & \text{for} \quad \sqrt[3]{\dfrac{3}{8}} \le x \le 4 \\ \left(\dfrac{x^3+x-3}{8x^3-3}\right)^{1/3} & \text{for} \quad x < \sqrt[3]{\dfrac{3}{8}} \end{cases}, \text{ equals}$$

to

(a) $-\frac{1}{2}$

(b) 1

(c) 0

(d) $\frac{3}{8}$

(e) $-\frac{1}{3}$

$$\lim_{x \to +\infty} \frac{2+\sqrt{x}}{2-\sqrt{x}} = -1$$

$$\lim_{x \to -\infty} \sqrt[3]{\frac{x^3+x-3}{8x^3-3}} = \frac{1}{2}$$

Q6.

An equation of the tangent line to the curve $y = \frac{1}{x^3}$ when $x = -1$ is given by

(a) $y = -3x - 4$

(b) $y = -2x - 3$

(c) $y = -\frac{1}{3}x - \frac{4}{3}$

(d) $y = \frac{1}{3}x + 1$

(e) $y = -3x - 2$

$y = x^{-3}$

$y' = -3x^{-4}$

$y'(-1) = -3$

$y(-1) = -1$

Q7.

The values of a and b for which the function

$$f(x) = \begin{cases} a & , \quad x \le 1 \\ \ln(x^4 - 1) - \ln(x - 1) & , \quad 1 < x \le 3 \\ \ln b & , \quad 3 < x \end{cases}$$

is continuous everywhere are:

(a) $a = 2\ln 2,\ b = 40$

(b) $a = \ln 2,\ b = \ln 40$

(c) $a = \ln 2,\ b = 20$

(d) $a = \ln 2,\ b = 10$

(e) $a = 20,\ b = \ln 3$

$f(1) = a$

$\lim_{x \to 1^+} f(x) = \lim_{x\to 1^+} \ln(x^4 - 1) - \lim \ln(x-1)$

$= \cancel{\ln 0^+ - \ln}$

$= \lim_{x \to 1^+} \ln \frac{x^4 - 1}{x - 1}$

$= \lim_{x \to 1^+} \ln \frac{(x-1)(x^3 + x^2 + x + 1)}{x - 1}$

$= \lim_{x \to 1^+} \ln x^3 + x^2 + x + 1$

$= \ln 4$

$\boxed{a = \ln 4}$

$= 2\ln 2$

$f(3) = \ln(3^4 - 1) - \ln(2)$

$= \ln \frac{81 - 1}{2} = \ln 40$

$\boxed{\ln b = \ln 40}$

Q8.

The function $g(x) = \dfrac{\sqrt{x^2 - 4}}{x^2 - 3x}$ is **continuous** on

(a) $(-\infty, -2] \cup [2, 3) \cup (3, \infty)$ ✓

(b) $(-\infty, \infty)$

(c) $(-\infty, 0) \cup (0, 3) \cup (3, \infty)$

(d) $(-\infty, -2] \cup [2, \infty)$

(e) $[-2, 0) \cup (0, 2]$

$x^2 - 4 \ge 0 \qquad \mathbb{R} - (-2, 2)$

$x(x - 3) \ne 0$

$x \ne 0$ or $x \ne 3$

$(-\infty, -2] \cup [2, 3) \cup (3, \infty)$

Q9.

If $f(x) = \begin{cases} \ln(x-5), & 5 < x \leq 6 \\ [\![x]\!] + [\![-x]\!], & x > 6 \end{cases}$. Find $\lim_{x \to 6} f(x)$.

$6-5=1 \quad \ln 1 = 0$

(a) Does not exist

(b) 0

(c) -1

(d) -2

(e) 1

$\lim_{x \to 6^-} \ln(x-5) = 0$

$\lim_{x \to 6^+} [\![x]\!] + [\![-x]\!] = 6-7 = -1$

$x \to 6^+ \qquad x = \underline{6+\delta}$

$-x = -(6+\delta)$

$[\![-(6+\delta)]\!] = -7$

Q10.

If $x^2 + 1 \leq f(x) - 2x \leq 3x^4 - 1$ for all $x \in (-\infty, \infty)$, then $\lim_{x \to 1} f(x) =$

by squeezing tho

(a) 4

(b) -3

(c) 1

(d) 0

(e) Does not exist

$\lim_{x \to 1} x^2+1 = 2 \quad \& \quad \lim_{x \to 1} 3x^4 - 1 = 2$

$\Rightarrow \lim_{x \to 1} f(x) - 2x = 2$

$\Rightarrow \lim_{x \to 1} f(x) - \lim_{x \to 1} 2x = 2$

$\Rightarrow \lim_{x \to 1} f(x) = 4$