Solution MATH-101-10 term 171 Quiz 2

Name:- ID:-

Q1. Evaluate the limit, if it exists:

a) $\lim_{x\to 3}[\![\frac{x}{2}+1]\!]$

$\lim_{x\to 3} f(x) = [\![\frac{3}{2}+1]\!] = [\![\frac{5}{2}]\!] = 2.$ (1)

b) $\lim_{x\to 0} x^2(1+\sin(x+1))$

$-1 \le \sin(x+1) \le 1$

$0 \le 1+\sin(x+1) \le 2$

$0 \le x^2(1+\sin(x+1)) \le 2x^2$

by squeezing th$^{\circ}$ (1)

$\lim_{x\to 0} x^2(1+\sin(x+1)) = 0$

$\lim_{x\to 0} 0 = 0 = \lim_{x\to 0} 2x^2$

Q2. Find all VA and HA for $f(x) = \dfrac{x^2-7x+12}{x^2+2x-15} = \dfrac{(\cancel{x-3})(x-4)}{(\cancel{x-3})(x+5)}$

the curve hase a hole at $x=3$ No V.A. (2)

$\boxed{x=-5 \text{ is V.A.}}$ $\lim_{x\to -5^-} f(x) = \infty$ & $\lim_{x\to -5^+} f(x) = -\infty$

& $\lim_{x\to\infty} f(x) = 1$ $\boxed{y=1 \text{ is H.A.}}$

Q3. Let $f(x) = \begin{cases} \dfrac{6c}{x+1} & \text{if } x>1 \\ cx+4 & \text{if } x\le 1 \end{cases}$ Find the value of c so that $f(x)$ continuous at $x=1$.

$f(1) = c+4 = \lim_{x\to 1^+} \dfrac{6c}{x+1} = \dfrac{6c}{2}$ (2)

$\Rightarrow 2c+8 = 6c \Rightarrow c=2$

Q4. Use the intermediate value theorem to show that the equation $3\sin x = 5\cos x$ has a solution in the interval $[0, \frac{\pi}{2}]$

$f(x) = 3\sin x - 5\cos x$

$f(0) = -5 \quad f(\frac{\pi}{2}) = 3$

by IMVT.

$\exists\, c \in (0, \frac{\pi}{2})$ such that

$f(c) = 0$

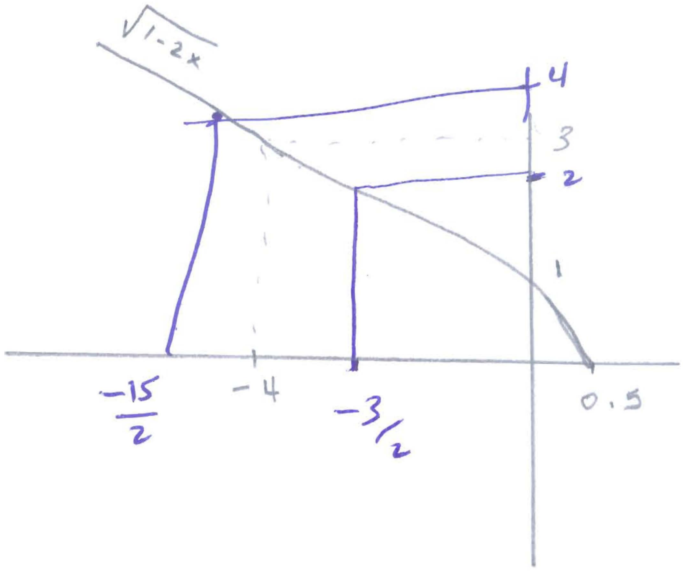
$3\sin c = 5\cos c$

Q5. If $\lim_{x\to -4} \sqrt{1-2x} = 3$. Find $\delta > 0$ such that, for all x satisfies $0 < |x + 4| < \delta$, the inequality $\left|\sqrt{1-2x} - 3\right| < \varepsilon$ holds, where $\varepsilon = 1$.

$\sqrt{1-2x} = 4$

$1 - 2x = 16$

$2x = -15 \quad x = -\frac{15}{2}$



$\sqrt{1-2x} = 2$

$1 - 2x = 4$

$-3 = 2x$

$x = -3/2$

$\delta_1 = -4 - (-\frac{15}{2}) = -(\frac{8-15}{2}) = 7/2$

$\delta_2 = \frac{-3}{2} - (-\frac{8}{2}) = 5/2$

$\delta = \min = 5/2$.