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Q Consider the function $f(x) = x^{\frac{1}{3}}(x+4)$ Follow the steps to sketch the Graph of the function.

- 1) Find y-int. then x-int. then check if the graph above the x-axis or below.

$$(0,0) \quad (-4,0)$$

$$f \begin{array}{ccccc} + & | & - & | & + \\ -4 & & 0 & & \end{array}$$

- 2) Find critical points then check if the graph increasing or decreasing, then find relative(local) extreme

$$\begin{aligned} f'(x) &= \frac{1}{3}x^{-\frac{2}{3}}(x+4) + x^{\frac{1}{3}} \\ &= \cancel{x}^{-\frac{2}{3}} \left[\frac{x+4+x}{3} \right] \\ &= x^{-\frac{2}{3}} \left(\frac{4}{3}x + \frac{4}{3} \right) \\ &= \frac{4(x+1)}{3\sqrt[3]{x^2}} \end{aligned}$$

$$\begin{array}{ccccc} - & | & + & | & + \\ \downarrow & & \downarrow & & \downarrow \\ c.p & 0, -1 & & & \end{array}$$

- 1) 3) Check the behavior of the graph as $x \rightarrow \infty$ and $x \rightarrow -\infty$
4) Find asymptotes if any

$$\lim_{x \rightarrow \infty} f(x) = \infty \quad \lim_{x \rightarrow -\infty} f(x) = \infty$$

NO Asymptotes

- 2) 5) Check if the graph concave up or down then find inflection points if any

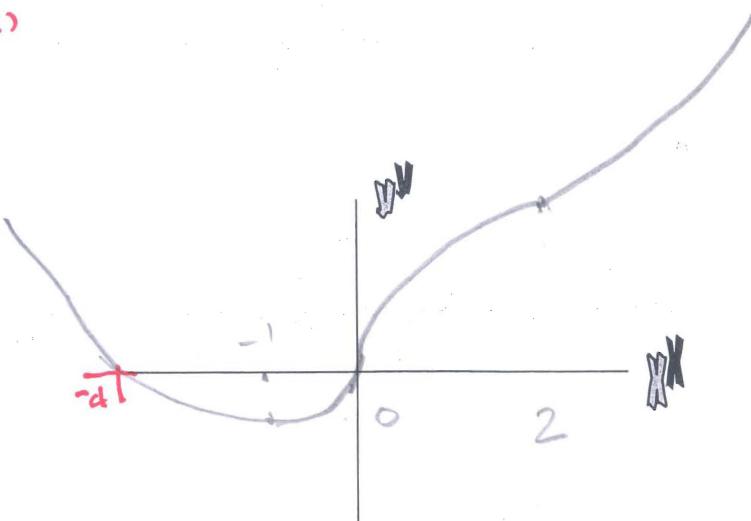
$$\begin{aligned} f'' &= \frac{4}{3} \left(-\frac{2}{3} \right) x^{-\frac{5}{3}}(x+1) + \frac{4}{3} x^{-\frac{2}{3}} \\ &= -\frac{4}{3} x^{\frac{5}{3}} \left[-\frac{2}{3}(x+1) + x \right] = \frac{4}{3} x^{-\frac{5}{3}} \left(\frac{1}{3}x - \frac{2}{3} \right) \end{aligned}$$

$$\begin{array}{ccccc} U & | & \cap & | & U \\ . & 0 & & 2 & + \end{array}$$

- 2) 6) Is there a vertical tangent $= \frac{4}{9}x^{-\frac{5}{3}}(x-2)$

$$\lim_{x \rightarrow 0^+} f(x) = \infty = \lim_{x \rightarrow 0^-} f(x)$$

V.T at $x=0$



- 2) 7) Sketch the graph