MATH-101-10 term 171 Quiz 4

Name:- ID:-

Q1. *If* $h(x) = xe^x$ and n is a positive integer, then find $f^{(n)}(1)$

$h'(x) = e^x + xe^x$

$h''(x) = e^x + e^x + xe^x = 2e^x + xe^x$

$h'''(x) = 2e^x + e^x + xe^x = 3e^x + xe^x$

$h^{(n)}(x) = ne^x + xe^x = h^{(n)}(x)$, Now $h^{(n)}(1) = ne + e$

$= \underline{\underline{(n+1)e}}$

Q2. Let f and g be differentiable functions and $h(x) = f(x^2 g(x))$ and if $g(2) = 0.5$, $f'(2) = 2$ and $g'(2) = 2$ then find $h'(2)$

$h'(x) = f'(x^2 \cdot g(x)) \cdot [2x\,g(x) + x^2 g'(x)]$

$h'(2) = f'(2^2 g(2)) \cdot [2 \cdot 2\, g(2) + 2^2 g'(2)]$

$= f'(4 \tfrac{1}{2}) \cdot [4 \tfrac{1}{2} + 4 \cdot 2] = f'(2)(10) = 2 \cdot (10) = 20$

Q3. Find $\lim_{x \to 0} \left(\dfrac{\sin x}{x + \tan x} \right)$

$$\lim_{x \to 0} \frac{\frac{\sin x}{x}}{\frac{x}{x} + \frac{\tan x}{x}} = \frac{\lim_{x \to 0} \frac{\sin x}{x}}{1 + \lim_{x \to 0} \frac{\sin x}{x} \cdot \lim_{x \to 0} \frac{1}{\cos x}}$$

$$= \frac{1}{1 + 1 \cdot 1} = \frac{1}{2}$$

Q4. The equation of the normal line to the graph of the curve $y = \dfrac{1 + \sin x}{x + \cos x}$ at $(\pi, \dfrac{1}{\pi - 1})$ is

Point given $(\pi, \frac{1}{\pi - 1})$ $y(\pi) = \frac{1+0}{\pi + (-1)} = \frac{1}{\pi - 1}$

Slope $y' = \dfrac{\cos x\,(x + \cos x) - (1 + \sin x)(1 - \sin x)}{[x + \cos x]^2}$

$y' = \dfrac{x \cos x + \cos^2 x - [1 - \sin^2 x]}{[x + \cos x]^2} = \dfrac{x \cos x}{(x + \cos x)^2}$; $y'(\pi) = \dfrac{\pi(-1)}{(\pi - 1)^2}$

$\cos^2 x$

m slope of tangent $= \dfrac{-\pi}{(\pi - 1)^2} \Rightarrow$ Slope of the normal line $\dfrac{(\pi - 1)^2}{\pi}$

then Equation of the normal line $y = \dfrac{(\pi - 1)^2}{\pi}(x - \pi) + \dfrac{1}{\pi - 1}$