

3.7 Rate of change in the natural.

$$\Delta x = x_2 - x_1$$

$$\Delta y = f(x_2) - f(x_1)$$

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x}$$

Motion: object position function at time t

$s(t)$ along the x -axis

$$\Delta s = s(t + \Delta t) - s(t)$$

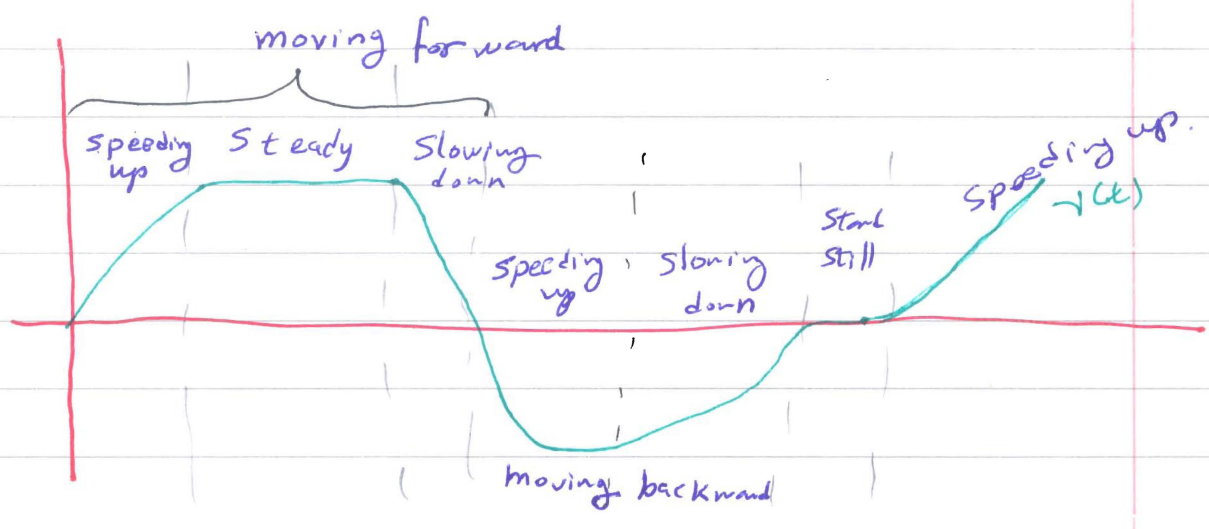
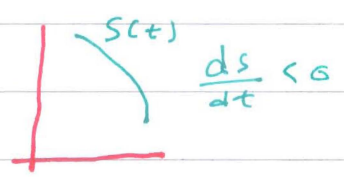
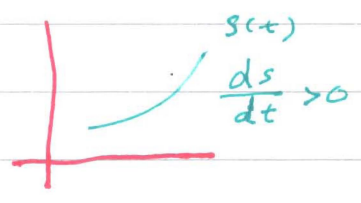
$$v_a = \Delta s / \Delta t \quad v(t) = \frac{ds}{dt}$$

speed = $|v(t)| = \left| \frac{ds}{dt} \right|$

Acceleration $a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$

Jerk $j(t) = \frac{da}{dt} = \frac{d^3s}{dt^3}$

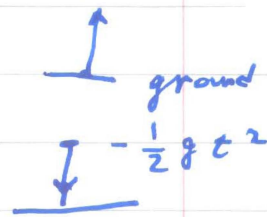
Object fall $s(t) = \frac{1}{2}gt^2; v = gt \quad a = g = 32 \text{ ft/s}^2$



Ex. A ball is thrown down with an initial speed of 20 ft/s from the top of a cliff, and it strikes the ground at the bottom of the cliff after 5s.
How high is the cliff?

Ans

$S(t) = \frac{1}{2} g t^2$ that is



$S(t) = -\frac{1}{2} g t^2 + v_0 t + S_0$

$0 = S(5) = -\frac{1}{2} 32(5)^2 - 20(5) + S_0 \Rightarrow S_0 = \underline{500}$

Ex. ball released $t=0$

• How many feet does the ball fall in 2s.

$S(2) = \frac{1}{2} 32(2)^2 = 64 \text{ ft.}$

• find velocity t .

$v(t) = 32t$

$S = \frac{1}{2} 32 t^2 \text{ ft}$

$= \frac{1}{2} 9.8 t^2 \text{ m}$

$v = 9.8 t$

Ex. let $S(t) = t^3 - 6t^2 + 9t$

when the object rest $v(t) = 0$

$v(t) = 3t^2 - 12t + 9 \quad t = 1, t = 3$

When moving forward $v(t) > 0$

$a(t) = 6t - 12$

$(0, 1) \cup (3, \infty)$ F

Total distance

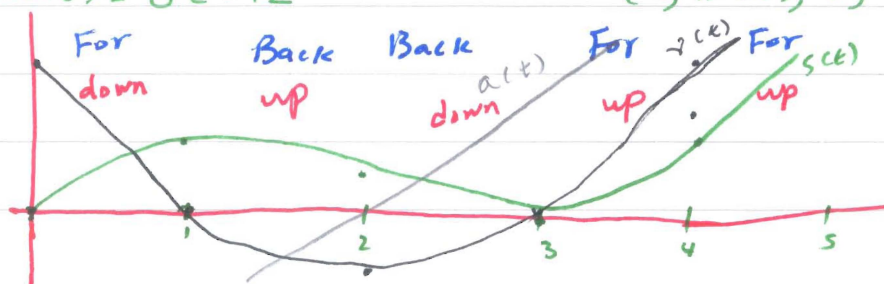
$t=0, t=2$

$|S(1) - S(0)| = 4$

$+ |S(2) - S(1)| = 2$

$\underline{6}$

$(1, 3)$ B



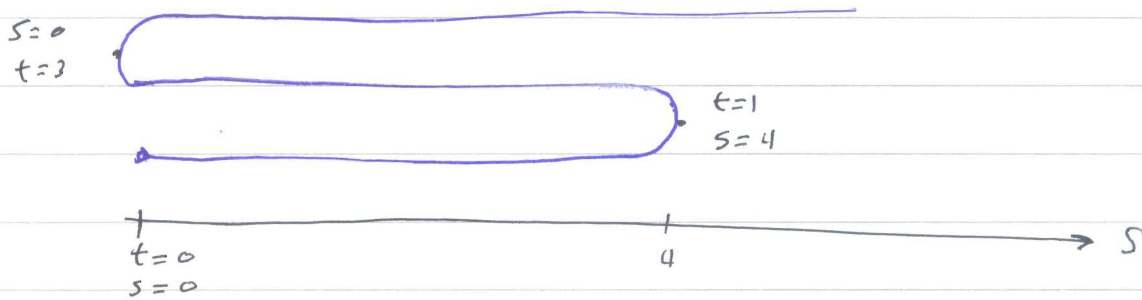
$v + \text{dec}$ D

$v - \text{dec}$ U

$v - \text{inc}$ D

$v + \text{inc}$ U

$$y(t) = 3t^2 - 6t + 9 = 3(t-1)(t-3)$$



Ex $s(t) = \sin \pi t + \cos \pi t \quad t > 0$

Find the total distance $0 \leq t \leq 1$

$$v(t) = \pi \cos \pi t - \pi \sin \pi t = 0$$

$$\cos \pi t = \sin \pi t \quad \pi t = \frac{\pi}{4} + k\pi \quad t = \frac{1}{4} + k$$

