3.4 The chain Rule

How to find the derivative $f \circ g$

Ex. $y = \sin(3x)$

$$y' = \cos(3x) \cdot \underline{3}$$

$y = f \circ g = \sin(g(x)) \quad g(x) = 3x$

$y = f(g(x))$

$$\frac{dy}{dx} = \frac{df}{dg} \cdot \frac{dg}{dx}$$

Th. If $f(u)$ is diff. at $u = g(x)$ and $g(x)$ is diff at x, then the composite $f \circ g(x) = f(g(x))$ is differentiable at x

$$(f \circ g)'(x) = f'(g(x))\, g'(x)$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Ex.

① $f(x) = (3x^2 + 2x - 1)^6$

$$y' = 6(3x^2 + 2x - 1)^5 \cdot (6x + 2)$$

② $f(x) = \sin x^3$

$$f'(x) = \cos x^3 \cdot 3x^2$$

③ $f(x) = \sec(u(x))$

$$f' = \sec u(x) \cdot \tan(u(x)) \cdot \frac{du}{dx}$$

$$\frac{d}{dx} u^n = n u^{n-1} \frac{du}{dx} \; ; \quad \frac{d[g(x)]^n}{dx} = n[g(x)]^{n-1} \cdot \frac{dg(x)}{dx}.$$

4) $y = \sin^3 x$

$$f(u) = u^3 \qquad u = \sin x$$

$$\frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx} = 3u^2 \cdot \cos x$$

5) $f(x) = \cos^2(3\sqrt{x})$

$$f' = 2\cos(3\sqrt{x}) \cdot (-\sin 3\sqrt{x}) \cdot \frac{3}{2\sqrt{x}}$$

6) $f(x) = \sqrt{\cos 5x}$

$$f' = \frac{1}{2}(\cos 5x)^{-\frac{1}{2}} \cdot 5$$

7) $f(x) = \sqrt{x} \tan^3 \sqrt{x}$

$$f' = \frac{1}{2\sqrt{x}} \tan^3\sqrt{x} + \sqrt{x}\; 3\tan^2\sqrt{x} \cdot \sec^2\sqrt{x} \cdot \frac{1}{2\sqrt{x}}$$

8) $f(x) = (1 + \sin^3(x^5))^{12}$

$$f' = 12(1 + \sin^3(x^5))^{11} \cdot (3\sin^2 x^5 \cdot \cos x^5 \; 5x^4)$$

9) Find the eq. of tangent line of $y = \sec^3(\frac{\pi}{2} - x)$ at $x = -\pi/2$.

$$y' = -3\sec^2(\tfrac{\pi}{2} - x) \cdot \sec(\tfrac{\pi}{2} - x)\tan(\tfrac{\pi}{2} - x)$$

$$y'(-\tfrac{\pi}{2}) = 0 \qquad y(-\tfrac{\pi}{2}) = -1$$

$$\boxed{y = -1}$$

(10) Find A so that $y = A\sin 3t$ satisfies the equation $\frac{d^2y}{dx^2} + 2y = 4\sin 3t$.

$y' = 3A\cos 3t \qquad y'' = -9A\sin 3t$

$-9A\sin 3t + 2A\sin 3t = 4\sin 3t$

$-9A + 2A = 4$

$A = -4/7.$

(11) $f(x) = \begin{cases} x\sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$ find f' if it exist

$f(x)$ cont at $x=0$ but not diff at $x=0$

Using sequeezing th°

$f'(0) = \lim_{h\to 0} \frac{h\sin\frac{1}{h} - 0}{h}$

DNE

$x \neq 0 \qquad f'(x) = \sin\frac{1}{x} + x\left(-\frac{1}{x^2}\right)\cos\frac{1}{x}$

(12) Given $f'(0) = 2$, $g(0) = 0$, $g'(0) = 3$

find $(f \circ g)'(0) = f'(g(0))\,g'(0) = 6$

$$\frac{d}{dx} a^x = \ln a \; a^x$$

Ex. $\frac{d}{dx} a^{2x} = \ln a \; a^{x} \; (2)$

Ex. $\frac{d}{d\theta} e^{\tan 3\theta} = e^{\tan 3\theta} \cdot 3\sec^2 3\theta$