3.3 Derivative of trigonometric functions.

$$\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1 \qquad \lim_{\theta\to 0}\frac{\cos\theta-1}{\theta}=0$$

Ex. $\lim_{x\to 0}\frac{\sin 2x}{5x}=\lim_{x\to 0}\frac{2}{5}\frac{\sin 2x}{2x}=\frac{2}{5}.$

Ex $\lim_{x\to 2}\frac{\cos \pi/x}{x-2}\left(\frac{0}{0}\right)=\lim_{t\to 0}\frac{\cos\left(\frac{\pi}{2}-t\right)}{\frac{4t}{\pi-2t}}$ put $t=\frac{\pi}{2}-\frac{\pi}{x}$ then $x\to 2 \Rightarrow t\to 0$

$$=\lim_{t\to 0}\frac{1}{4}\frac{\sin t}{t}\cdot\lim_{t\to 0}(\pi-2t)$$

$$=\pi/4.$$

or $\lim_{x\to 2}\frac{\sin\left(\frac{\pi}{2}-\frac{\pi}{x}\right)}{x-2}=\lim_{x-2\to 0}\frac{\pi/2x}{}\ \frac{\sin\frac{\pi}{2x}(x-2)}{x-2}=\pi/4.$

Ex. $\lim_{x\to 0}\frac{2x+\sin 3x}{x}=5.$ Ex. $\lim_{x\to a}\frac{x^2-a^2}{\sin x+a}=2a.$

Ex. $\lim_{t\to 0}\frac{\cos^2 t-2\cos t+1}{t^2}=\lim_{t\to 0}\frac{(\cos t-1)^2}{t^2}$

$$=\left(\lim_{t\to 0}\frac{\cos t-1}{t}\right)^2$$

$$=0.$$

$\frac{d}{dx}\sin x = \cos x$ $\quad$ $\frac{d}{dx}\cos x = -\sin x$

$\frac{d}{dx}\tan x = \sec^2 x$ $\quad$ $\frac{d}{dx}\cot x = -\csc^2 x$

$\frac{d}{dx}\sec x = \sec x \tan x$ $\quad$ $\frac{d}{dx}\csc x = -\csc x \cot x$.

Sin Rule

Ex.

① $y = \tan x$ prove $y' = \sec^2 x$

② $y = \sec x \tan x$

$$\frac{dy}{dx} = \sec x \tan^2 x + \sec^3 x$$

③ $f(x) = 2\cos x - 3\sin x$

$f'(x) =$

④ $y(\theta) = \dfrac{\csc\theta}{\tan\theta}$ $\quad$ $y' = \dfrac{-\csc\theta\cot\theta\tan\theta - \csc\theta\sec^2\theta}{\tan^2\theta}$

$$= \frac{-\csc\theta(1+\sec^2\theta)}{\tan^2\theta}$$

⑤ Find the equation of the line tangent to the graph of $\cos x$ at the point $x = \pi/2$.

$x = \pi/2$ $\quad y = 0$

$m = -\sin \pi/2 = -1$

Eq $\rightarrow$ $y = -x + \pi/2$

⑥ $y = \dfrac{\tan x}{1+\tan x}$ $\quad$ $y' = \dfrac{\sec^2 x}{(1+\tan x)^2}$ $\quad$ (+)

(7) Find $\frac{d^{23}}{dx^{23}}(x \sin x)$

Ans $y = x \sin x$

y'

y''

$f(x) = \sin(3x - bt)$

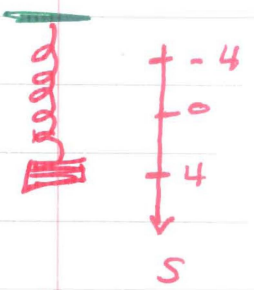
$f^{(x)} = \begin{cases} (-1)^k 3^k \sin & k \text{ even} \\ (-1)^k 3^k \cos & k \text{ odd} \end{cases}$

Nex section
Chain Rule

Patrron

(8) An object at the end of a vertical spring is stretched 4 Cm beyond its rest position and released at time $t=0$

Find the velocity at time t and use it to analyze the motion of the object



The position function

$s(t) = 4 \cos t.$

Ans

$$v = \frac{ds}{dt} = -4 \sin t$$

The object ocsillates between -4 & 4, the period of oscillation is 2π (cost)

The speed $|v| = 4|\sin t|$

max when $|\sin t| = 1$ $t = \pi/2, -\pi/2$

fastest (max velocity) $t=0, s=0$

Speed $= 0$, $\sin t = 0$ $t = 0$