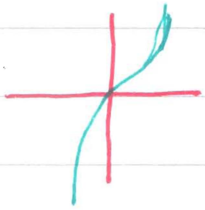
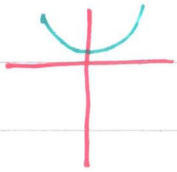
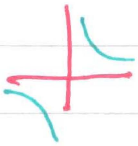
3.11 Hyperbolic functions

$\sinh = \dfrac{e^{x} - e^{-x}}{2}$ $\qquad$ $\cosh = \dfrac{e^{x} + e^{-x}}{2}$

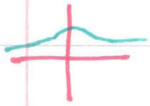




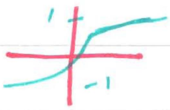
csch x



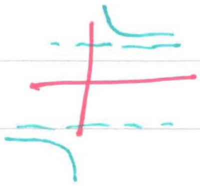
sech x



tanh x



coth x



Identities $\cosh^2 x - \sinh^2 x = 1$ $\qquad$ $\cosh(-x) = \cosh x$

$1 - \tanh^2 x = \operatorname{sech}^2 x$ $\qquad$ $\sinh(-x) = -\sinh x$

$\sinh x + y = \sinh x \cosh y + \cosh x \sinh y$

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Ex.

$\sinh x = -3/4$ find the rest.

$\cosh x = \sqrt{1 + \sinh^2 x} = 5/4$ $\qquad$ $\tanh x = \dfrac{\sinh x}{\cosh x} = -3/5$

$\coth x = -5/3$ $\quad$ $\operatorname{sech} x = 4/5$ $\quad$ $\operatorname{csch} x = -4/3$

Ex Rewrite in term of e^x & simplify

ⓐ $\sinh(2 \ln x) = \dfrac{x^2 - \frac{1}{x^2}}{2} = x^4 - 1/2x^2$

Ex ⓑ $\cosh 3x - \sinh 3x = e^{-3x}$

ⓒ $\ln(\cosh x + \sinh x) + \ln(\cosh x - \sinh x)$

$\ln(ab) = \ln(\cosh^2 x - \sinh^2 x) = \ln 1 = 0$

Ex. Show $\cosh^2 x - \sinh^2 x = 1$

Derivative

$\frac{d}{dx}\sinh x = \cosh x$ $\quad$ $\frac{d}{dx}\cosh x = \sinh x$

$\frac{d}{dx}\tanh x = \sec^2 h$ $\quad$ $\frac{d}{dx}\coth x = -\csc^2 h$

$\frac{d}{dx}\operatorname{sech} x = -\operatorname{sech} x \tanh x$ $\quad$ $\frac{d}{dx}\operatorname{csch} = -\operatorname{csch} x \coth x.$

Ex. find y' ⓐ $y = \ln\cosh(z)$ $\quad$ $\frac{dy}{dz} = \tanh z$

ⓑ $\ln\cosh v - \frac{1}{2}\tanh^2 v$

$$\frac{dy}{dv} = \tanh v - \tanh v\,(\operatorname{sech}^2 v)$$
$$= \tanh v\,(1 - \quad) = \tanh^3 v.$$

ⓒ $y = (x^2+1)\operatorname{sech}(\ln x)$ $\quad$ $\frac{dy}{dx} = 2$

$$\frac{dy}{dx} = 2x\operatorname{sech}(\ln x) + \frac{(x^2+1)\cdot\cosh(\ln x)}{x}$$

$$y = \frac{(x^2+1)\,2}{e^{\ln x} + e^{-\ln x}} = \frac{(x^2+1)\,2x}{x^2+1} = 2x \quad \frac{dy}{dx} = 2.$$

ⓓ $y = \operatorname{csch} 2^{\theta}$ $\quad$ $\frac{dy}{d\theta} = -\operatorname{csch} 2^{\theta}\coth 2^{\theta}\cdot \ln 2\, 2^{\theta}$