3.1 Derivatives of Polynomials and Exponential function.

$\boxed{1}$ $f(x) = c$ then $\frac{df}{dx} = \frac{dc}{dx} = 0$ Contant Rule

How easy

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{c-c}{h} = 0$$

power function

$\boxed{2}$ $f(x) = x$ $\frac{df}{dx} = \frac{dx}{dx} = 1$

Also easy

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{x+h-x}{h} = 1$$

Ex. $\frac{d}{dx} x^2 = 2x$ $\frac{d}{dx} x^3 = 3x^2$

$\boxed{3}$ If n is positive, the

$$\frac{d}{dx} x^n = n x^{n-1}$$

Using Binomial th°.

Ex. $\frac{d}{dr} r^\pi = \pi r^{\pi-1}$. Ex. $\frac{d}{dx} \frac{1}{x^3} = \frac{d}{dx} x^{-3} = -3x^{-4}$
$= \frac{-3}{x^4}$

$\boxed{3}'$ In general

$$\frac{d}{dx} x^n = n x^{n-1} \quad \text{for any } n \in \mathbb{R}$$

Ex. $y = \sqrt{x}$ $\frac{dy}{dx} = \frac{d}{dx} x^{1/2} = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$

4 The constant multiple Rule

If c is a constant and f is a differentiable function, then

$$\frac{d}{dx} c f(x) = c \frac{df}{dx}.$$

easy.

5 The sum Rule

If f and g are both diff, then

$$\frac{d}{dx}\left(f(x) \underline{+} g(x)\right) = \frac{d}{dx} f(x) \underline{+} \frac{d}{dx} g(x)$$

easy

6 The difference Rule

Ex. The equation of a motion is $s = 2t^3 + 3t - 4$ measured in cm. find the accelaration function $a(t)$ and the the accelaration at 2 seconds

$$v(t) = \frac{ds}{dt} = 6t^2 + 3$$

$$a(t) = \frac{dv}{dt} = 12t \qquad a(2) = 24 \text{ cm}/s^2$$

Exponential functions

$f(x) = b^x$

by definition

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{b^{x+h} - b^x}{h}$$

$$= \lim_{h\to 0} \frac{b^x(b^h - 1)}{h} = b^x \lim_{h\to 0} \frac{b^h - 1}{h}$$

Now $f'(0) = b^0 \lim_{h\to 0} \frac{b^h-1}{h} = \lim_{h\to 0} \frac{b^h-1}{h}$

So $f'(x) = b^x f'(0)$

we can prove that $f'(0) = \ln b$

$$f'(x) = b^x \ln b.$$

Now for the base e (replace b by e)

we can prove that

$$f'(0) = \lim_{h\to 0} \frac{e^h - 1}{h} = 1$$

So $\frac{d}{dx} e^x = e^x$

Ex. when $y = e^x$ (at what point) parallel to the line $y = 2x$