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2.7 Derivative and Rate of Change.

from section 2.1: The slope of the Secant

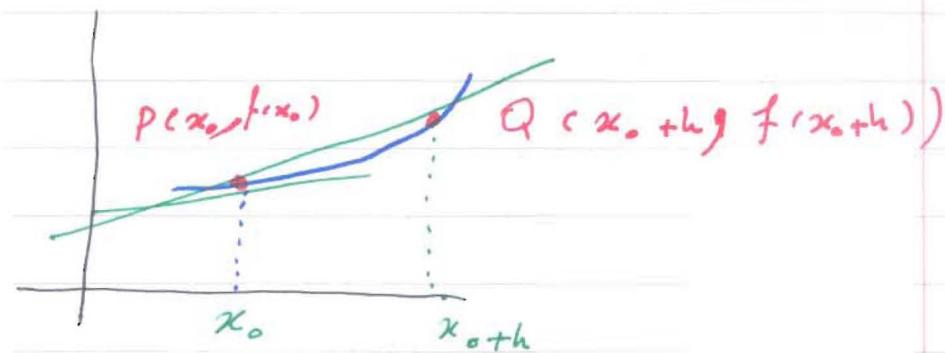
$$\frac{\Delta y}{\Delta x} = \frac{f(x_0+h) - f(x_0)}{h}$$

Tangent

↓
velocity

derivative

Rate of change



Def. The tangent line to the curve $y=f(x)$ at the point $P(a, f(a))$ is the line through P with slope

$$m = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

$$a = x_0 \quad x = x_0 + h \quad \text{then}$$

$$m = \lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

$$\begin{aligned} & 3x^2 - 3 \\ & (3x^2 - 1)(x + a) \\ & \alpha 3x - x \end{aligned}$$

$$\begin{aligned} & \cancel{y = 3x^2} \\ & x = 3 \\ & y(3) = 36 \end{aligned}$$

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Ex. find the equation of the tangent line
to $y = x^2$ at $P(1, 1)$

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = \lim_{x \rightarrow 1} x + 1 = 2.$$

Ex. $y = \frac{3}{x}$ at $(3, 1)$

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{\frac{3}{x+h} - \frac{3}{x}}{h} &= \lim_{h \rightarrow 0} \frac{9 - (9+3h)}{3h(3+h)} \\ &= \lim_{h \rightarrow 0} \frac{-3h}{3h(3+h)} = -\frac{1}{3} \end{aligned}$$

velocities

Suppose an object moves along straight line $s = f(t)$ where s is the displacement at t .

f describes the motion called position function

the average velocity

$$v_a = \frac{f(a+h) - f(a)}{h}$$

Instantaneous velocity

$$v(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

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The instantaneous velocity

is the velocity at a precise moment.

Ex. During the first 40 s of a rocket flight the rocket is propelled straight up so that in t seconds it reaches a height of $s = 5t^3$ ft

(a) How high does the rocket travel in 40 s.

$$s = s(40) = 32,000 \text{ ft}$$

(b) what is the average velocity of the rocket during the first 40 s.

$$v_{av} = \frac{32,000}{40} = 8,000 \text{ ft/s}$$

(c) what is the average velocity of the rocket during the first 135 ft of its flight

$$s = 135 \text{ ft} = 5t^3 \quad t = 3 \text{ s.}$$

$$\text{Hence } v_{av} = \frac{135}{3} = 45 \text{ ft/s.}$$

(d) what is the instantaneous velocity of the rocket at the end of 40 s?

$$v_{ins} = \lim_{t_i \rightarrow 40} \frac{s(t_i^3 - 40)}{t_i - 40} \quad \lim_{t_i \rightarrow 40} (t_i^2 + 40t_i + 40^2) \\ = 24,000 \text{ ft/s}$$

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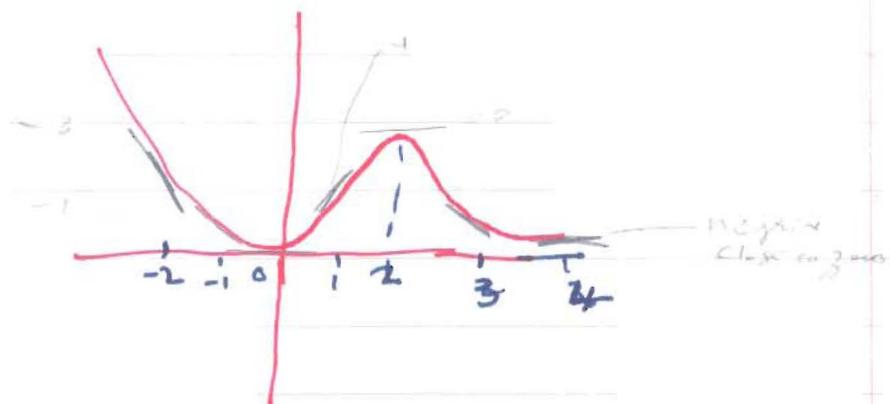
Ex. A particle moves in the positive direction along a straight line so that after t minutes its distance is $s = 6t^4$ feet from the origin. Find the average velocity of the particle over the interval $[2, 4]$.

$$\frac{s(4) - s(2)}{4 - 2} = \frac{6(4)^4 - 6(2)^4}{2}$$

Find the instantaneous velocity at $t=2$

$$\begin{aligned} v_{\text{inst}} &= \lim_{t \rightarrow 2} \frac{f(t) - f(2)}{t - 2} = \lim_{t \rightarrow 2} \frac{6t^4 - 6(2)^4}{t - 2} \\ &= \lim_{t \rightarrow 2} \frac{6(t-2)(t+2)(t^2+4)}{t-2} \\ &= 6(8)(4). \end{aligned}$$

Ex. Estimate the slope of the curve at all integer numbers given



Derivatives

Def. The derivative of a function f at a number a is

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

Ex. $f(x) = x^2 - 2x + 1$ at a find $f'(a)$

$$\begin{aligned} f'(a) &= \lim_{\substack{\text{lin} \\ h \rightarrow 0}} \frac{(a+h)^2 - 2(a+h) + 1 - a^2 + 2a - 1}{h} \\ &= \lim_{\substack{\text{lin} \\ h \rightarrow 0}} \frac{2ah + h^2 - 2h}{h} = \lim_{\substack{\text{lin} \\ h \rightarrow 0}} \frac{2a + h - 2}{1} = 2a - 2 \end{aligned}$$

def

The tangent line to $f(x)$ at a is

$$y - f(a) = f'(a)(x - a)$$

Rate of change.

x changes from x_1 to x_2 increment

$$\Delta x = x_2 - x_1$$

$$\Delta y = f(x_2) - f(x_1)$$

The difference $\frac{\Delta y}{\Delta x} =$

The instantaneous rate of change

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = f'(a) ?$$

The number M of milk production company locations
is given

Year	2001	2002	2003	2004
M	71	73	112	130

a) Find the average rate of growth

i) from 2001 to 2004

b) Sketch a graph and estimate Inst. RofC at 2001

at 2002
at 2003

a) average RofC $\frac{\Delta y}{\Delta x} = \frac{130 - 71}{3} = \frac{59}{3}$

b)

Instantaneous RofC

at 2001 = 0.01

2002 = 0.9

2003 = 1

