

3.6 Derivatives of logarithmic functions

$$f(x) = 3x + 1 \quad f'(x) = \frac{x-1}{3}$$

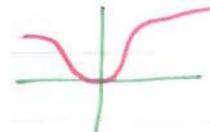
$$f' = 3 \quad (f')^{-1} = \frac{1}{3}$$

$$f \circ g(x) = x \quad g \circ f(x) = x$$

must be 1-1

restricted domain

Horizontal line to check



$$\text{Ex. } f(x) = y = (x-1)^3 + 2$$

$$f^{-1}(x) = \sqrt[3]{x-2} + 1$$

$f(x)$ & $f^{-1}(x)$ are symmetric on $y=x$.

$$\text{Ex. } f(x) = x^2 \quad f^{-1} = \sqrt{x} \quad \text{Domain } [0, \infty)$$

Th^o $f'(x)$ in I $f'(x)$ exist not zero

then f^{-1} diff in Domain

$$(f^{-1})'(b) = \frac{1}{f'(f^{-1}(b))}$$

(7.9)

$$f^{-1}(x)$$

last example

$$x=2 \quad f(2)=4$$

$$(f^{-1})'(c_1) = \frac{1}{f'(f^{-1}(4))} = \frac{1}{f'(c_2)} = \frac{1}{4}$$

Ex. $f(x) = x^3 - 2$ find $\frac{df^{-1}}{dx}$ at $x=6$
without finding $f'(x)$

$$\frac{df^{-1}}{dx}(6) = \frac{1}{f'(f^{-1}(6))} = \frac{1}{f'(c_2)}$$

$$f' = 3x^2 \quad f'(6) = 12$$

$$= \frac{1}{12}.$$

$$\frac{d}{dx} \ln u = \frac{1}{u} \frac{du}{dx}$$

$$\frac{d}{dx} \ln|x| = \frac{1}{x} \quad x \neq 0$$

$$\frac{d}{dx} a^u = a^u \ln a \frac{du}{dx}$$

$$\frac{d}{dx} \log_a x = \frac{1}{x \ln a} \quad u(n)?$$

$$\ln x^a = a \ln x$$

$$x = \ln ex = e^{\ln x}$$

$$e = \lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}}$$

$$= \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$$

Ex.

$$\textcircled{1} \quad y = (\ln x)^2$$

$$y' = \frac{2 \ln x}{x}$$

$$\textcircled{2} \quad y = \cos(\ln x)$$

$$y' = -\frac{\sin(\ln x)}{x}$$

$$\textcircled{3} \quad y = \ln \frac{\cos x}{\sqrt{4-3x^2}}$$

$$y' = \frac{\sec^2 x}{\tan x}$$

$$\textcircled{4} \quad y = \ln |\tan x|$$

$$\textcircled{5} \quad y = \ln |x|$$

$$y = \begin{cases} \ln x & x > 0 \\ \ln(-x) & x < 0 \end{cases}$$

$$y' = \frac{1}{x} \quad y' = \frac{-1}{-x} = \frac{1}{x}$$

both cases

$$\textcircled{6} \quad \ln \ln x$$

$$y = \frac{1}{x \ln x}$$

(81)

Logarithmic differentiation

Ex $y = x^x$

$$\ln y = \ln x^x = x \ln x$$

$$\frac{y'}{y} = \ln x + 1$$

Ex $y = \sqrt{\frac{x-1}{x+1}} \Rightarrow \ln y = \frac{1}{2} \ln(x-1) - \ln(x+1)$

$$\frac{y'}{y} = \frac{1}{2} \left(\frac{1}{x-1} - \frac{1}{x+1} \right)$$

Ex. $y = [\sin x]^{\ln x}$

$$\ln y = \ln x \ln \sin x$$

$$\frac{y'}{y} = \frac{\ln \sin x}{x} + \frac{\ln x \cdot \cos x}{\sin x}$$

Ex $y = \pi^{\sin x} \quad y' = \pi^{\sin x} \ln \pi \cos x$

Ex. $y = [(x + \tan y)]^x$

$$\frac{y'}{y} = \frac{x(\tan y + x \sec^2 y)}{x + \tan y}$$

$$\ln y = x \ln(x + \tan y)$$

$$+ \ln(x + \tan y)$$

find y' if

$$x = y^x$$

$$\Rightarrow y \ln x = x \ln y$$

$$\Rightarrow y' \ln x + y \frac{1}{x} = \ln y + \frac{x}{y}$$

The number e is a limit

$$e = \lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}}$$

$$= \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n.$$