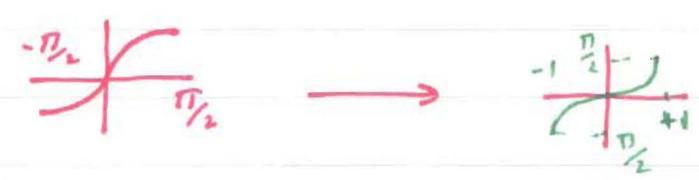


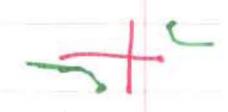
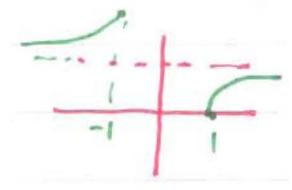
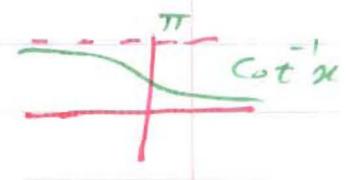
Derivative of Inverse Trigonometric function

$y = \sin x \Rightarrow y^{-1} = \sin^{-1} x = \arcsin x$
 Restrict Domain to $[-\pi/2, \pi/2]$ → Range $\sin^{-1} x$ Domain $[-1, 1]$



$\cos^{-1} x = \arccos x$	$D = [-1, 1]$	$R = [0, \pi]$
$\tan^{-1} x$	$D = \mathbb{R}$	$R = (-\pi/2, \pi/2)$
$\cot^{-1} x$	$D = \mathbb{R}$	$R = (0, \pi)$

$\sec^{-1} x$ $D = (-\infty, -1] \cup [1, \infty)$
 $R = [0, \pi] \setminus \{\pi/2\}$



$\csc^{-1} x$ $D = (-\infty, -1] \cup [1, \infty)$ $R = [-\pi/2, \pi/2] \setminus \{0\}$

$\sin^{-1} \sin x = x$ if $x \in [-\pi/2, \pi/2]$
 $\sin \sin^{-1} x = x$ if $x \in [-1, 1]$

Identities ① $\cos(\sin^{-1} x) = \sqrt{1-x^2}$

- ② $\sin(\cos^{-1} x) = \sqrt{1-x^2}$
- ③ $\tan(\sin^{-1} x) = x/\sqrt{1-x^2}$ ④ $\sec(\tan^{-1} x) = \sqrt{1+x^2}$
- ⑤ $\sin^{-1} x + \cos^{-1} x = \pi/2$ ⑥ $\sec^{-1} x = \cos^{-1} 1/x$
- ⑦ $\sin \sec^{-1}(x) = \sin \cos^{-1} 1/x$ ⑧ $\tan \sec^{-1} x = \frac{x}{|x|} \sqrt{x^2-1}$

Function	Domain	Range	Basic Relationships	equivalence.
$y = \sin^{-1} x$	$[-1, 1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$	$\sin(\sin^{-1} x) = x$ if $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ $\sin^{-1}(\sin x) = x$ if $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$	$\sin y = x$ if $x \in [-1, 1], y \in [-\frac{\pi}{2}, \frac{\pi}{2}]$
$y = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$	$\cos(\cos^{-1} x) = x$ if $x \in [-1, 1]$ $\cos^{-1}(\cos x) = x$ if $x \in [0, \pi]$	$\cos y = x$ if $x \in [-1, 1], y \in [0, \pi]$
$y = \tan^{-1} x$	\mathbb{R}	$(-\frac{\pi}{2}, \frac{\pi}{2})$	$\tan(\tan^{-1} x) = x$ if $x \in \mathbb{R}$ $\tan^{-1}(\tan x) = x$ if $x \in (-\frac{\pi}{2}, \frac{\pi}{2})$	$\tan y = x$ if $x \in \mathbb{R}, y \in (-\frac{\pi}{2}, \frac{\pi}{2})$
$y = \sec^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$	$\sec(\sec^{-1} x) = x$ if $x \in (-\infty, -1] \cup [1, \infty)$ $\sec^{-1}(\sec x) = x$ if $x \in [0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$	$\sec y = x$ if $x \geq 1$ or $x \leq -1$ $0 \leq y < \frac{\pi}{2}$ or $\frac{\pi}{2} < y \leq \pi$
$y = \cot^{-1} x$	\mathbb{R}	$(0, \pi)$	$\cot(\cot^{-1} x) = x$ if $x \in \mathbb{R}$ $\cot^{-1}(\cot x) = x$ if $x \in (0, \pi)$	$\cot y = x$ if $x \in \mathbb{R}, y \in (0, \pi)$
$y = \csc^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$[-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2}]$	$\csc(\csc^{-1} x) = x$ if $x \in (-\infty, -1] \cup [1, \infty)$ $\csc^{-1}(\csc x) = x$ if $x \in [-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2}]$	$\csc y = x$ if $x \geq 1$ or $x \leq -1$ $-\frac{\pi}{2} \leq y < 0$ or $0 < y \leq \frac{\pi}{2}$

Examples

① $\sin^{-1}(\sin \frac{\pi}{16}) = \frac{\pi}{16}$

② $\sin^{-1}(\sin \frac{9\pi}{5}) = \sin^{-1}(\sin(2\pi - \frac{\pi}{5})) = \sin^{-1}(\sin(-\frac{\pi}{5})) = -\frac{\pi}{5}$

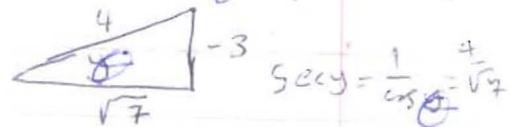
③ $\sin^{-1}(\sin \frac{7\pi}{3}) = \sin^{-1}(\sin(2\pi + \frac{\pi}{3})) = \sin^{-1}(\sin \frac{\pi}{3}) = \frac{\pi}{3}$

④ $\sin(\sin^{-1} \frac{1}{3}) = \frac{1}{3}$

⑤ $\sin(\sin^{-1} 2) =$ not defined. out of the domain.

⑥ $\sec[\sin^{-1}(-\frac{3}{4})] = \frac{4}{\sqrt{7}} \rightarrow \text{let } \sec \theta = b$

$\sin^{-1}(-\frac{3}{4}) = \theta \Rightarrow \sin \theta = -\frac{3}{4}$



$\sec \theta = \frac{1}{\cos \theta} = \frac{5}{4}$

$\theta = \cos^{-1}(\frac{4}{5})$

$\cos \theta = \frac{4}{5}$

⑦ $\sin(2 \cos^{-1}(\frac{3}{5}))$

$\sin 2\theta = 2 \sin \theta \cos \theta$

$= 2 \cdot \frac{4}{5} \cdot \frac{3}{5} = \frac{24}{25}$



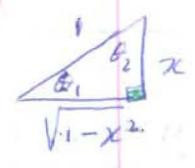
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$y = \sec^{-1} x = \cos^{-1}(\frac{1}{x}) ; x = \sec y \Rightarrow \cos y = \frac{1}{x}$
 Now $\sin \sec^{-1} x = \sin \cos^{-1}(\frac{1}{x})$
 $= \sqrt{1 - (\frac{1}{x})^2} = \frac{\sqrt{x^2 - 1}}{|x|}$

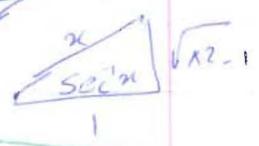
Triangle identities

$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$
 $\cos(\sin^{-1} x) = \sqrt{1-x^2}$
 $\sin(\cos^{-1} x) = \sqrt{1-x^2}$
 $\tan(\sin^{-1} x) = \frac{x}{\sqrt{1-x^2}}$

let $\theta_1 = \sin^{-1} x ; \theta_2 = \cos^{-1} x$



$\sec(\tan^{-1} x) = \sqrt{1+x^2}$
 $\sin(\sec^{-1} x) = \frac{\sqrt{x^2-1}}{|x|}$



Odd functions

$\sin^{-1}(-x) = -\sin^{-1} x$ $\tan^{-1}(-x) = -\tan^{-1} x$

Proof
 Why

$\theta = \sin^{-1}(-x) \Rightarrow -x = \sin \theta \Rightarrow \sin x = -\sin \theta$
 $\Rightarrow x = \sin(-\theta) = \sin^{-1} x$
 $\Rightarrow \theta = -\sin^{-1} x$

Derivatives

$[f'(x)]' = \frac{1}{f'(f^{-1}(x))}$

$y = \sin^{-1} x \Rightarrow \sin y = x \Rightarrow y' \cos y = 1$
 $\Rightarrow y' = \frac{1}{\cos y} = \frac{1}{\cos \sin^{-1} x} = \frac{1}{\sqrt{1-x^2}}$

$y = \sin^{-1} u(x)$
 $\frac{dy}{dx} = \frac{du/dx}{\sqrt{1-u^2}}$

in the same way

$\frac{d}{dx} (\cos^{-1} u(x)) = -\frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$

$\frac{d}{dx} (\tan^{-1} u) = \frac{1}{1+u^2} \frac{du}{dx}$

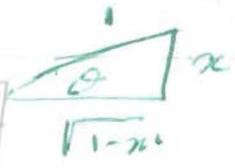
$\frac{d}{dx} (\sec^{-1} u) = \frac{1}{|u|\sqrt{u^2-1}} \frac{du}{dx}$

$\frac{d}{dx} (\cot^{-1} u) = -\frac{1}{1+u^2} \frac{du}{dx}$

$\frac{d}{dx} [\csc^{-1} u] = -\frac{1}{|u|\sqrt{u^2-1}} \frac{du}{dx}$

1) $\cos(\sin^{-1}x) = \sqrt{1-x^2}$

using $\sin^{-1}x = \theta \Rightarrow \sin\theta = x$

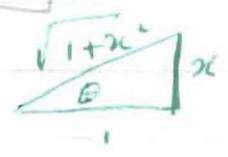


2) $\sin(\cos^{-1}x) = \sqrt{1-x^2}$

2b) $\tan \cos^{-1}x = \frac{\sqrt{1-x^2}}{x}$

3) $\tan(\sin^{-1}x) = \frac{x}{\sqrt{1-x^2}}$

4) $\sec(\tan^{-1}x) = \sqrt{1+x^2}$

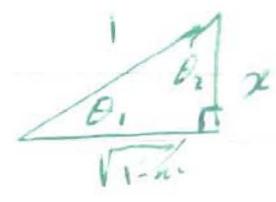


5) $\sin^{-1}x + \cos^{-1}x = \pi/2$

let $\theta_1 = \sin^{-1}x$ $\theta_2 = \cos^{-1}x$

$x = \sin\theta_1$ $x = \cos\theta_2$

$\theta_1 + \theta_2 = \pi/2$



6) $\sec^{-1}x = \cos^{-1}(1/x)$

$y = \sec^{-1}x \Rightarrow x = \sec y \Rightarrow \cos y = 1/x$

$\Rightarrow y = \cos^{-1}(1/x)$

7) $\sin \sec^{-1}x = \sin \cos^{-1}(1/x) \xrightarrow{\text{using 2i}} \sqrt{1 - (1/x)^2} = \frac{\sqrt{x^2-1}}{|x|}$

8) $\tan \sec^{-1}x \xrightarrow{6} \tan \cos^{-1}(1/x) \xrightarrow{2b} \frac{\sqrt{1-(1/x)^2}}{1/x} = \frac{x}{|x|} \sqrt{x^2-1}$

$\cos^{-1}x = \pi/2 - \sin^{-1}x$; $\cot^{-1}x = \pi/2 - \tan^{-1}x$; $\csc^{-1}x = \pi/2 - \sec^{-1}x$

3. Higher derivatives

~~$\frac{1}{x}$~~ $\sin x$

Examples

① $y = e^x \sec^{-1} x$

$$y' = e^x \sec^{-1} x + \frac{e^x}{x\sqrt{x^2-1}}$$

② $x^3 + x \tan^{-1} y = e^y$

$$3x^2 + \tan^{-1} y + x \frac{y'}{1+y^2} = y' e^y$$

③ $y = \sin^{-1}(\frac{1}{x})$

$$y' = \frac{1}{\sqrt{1-(\frac{1}{x})^2}} \left(-\frac{1}{x^2}\right) = \frac{-1}{x\sqrt{x^2-1}}$$

④ $y = \cos^{-1}(\cos x) \quad x \in [0, \pi]$

$$y' = \frac{-1}{\sqrt{1-\cos^2 x}} (-\sin x)$$

$$y' = \frac{\sin x}{|\sin x|} = 1 \quad x \in [0, \pi]$$

$\Rightarrow \sin x > 0$

Same

$y = \cos^{-1}(\cos x) = x \quad y' = 1$