1. X : heat evolved ; $X \sim N(\mu, 4)$
based on sample of size $n = 9$.
$H_0: \mu = 100$ vs $H_a: \mu \neq 100$

(i) Rejection Region is $(-\infty, 98.5] \cup [101.5, \infty)$

$\alpha = P(\text{Reject } H_0 \text{ when } H_0 \text{ is true})$

when H_0 is true $\bar{X} \sim N(\mu, \frac{4}{9})$

$\Rightarrow \alpha = P(\bar{X} < 98.5) + P(\bar{X} > 101.5)$

$= P\left(\frac{\bar{X}-100}{2/3} < \frac{98.5-100}{2/3}\right) + P\left(\frac{\bar{X}-100}{2/3} > \frac{101.5-100}{2/3}\right)$

$= P(Z < -2.25) + P(Z > 2.25)$

$= 0.0244$

(ii) $\beta = P(\text{type II error})$ when $\mu = 103$

$= P(\text{Accept } H_0 \text{ when false, i.e., } \mu = 103)$

$= P(98.5 < \bar{X} < 101.5)$

$= P\left(\frac{98.5-103}{2/3} < Z < \frac{101.5-103}{2/3}\right)$

$= P(-6.75 < Z < 2.25)$

$= 0.9878$

2. X: life (in hours) of a bulb; $X \sim N(\mu, \sigma^2)$; $\sigma = 25$ hrs

(i) Test $H_0: \mu = 1000$ vs $H_a: \mu > 1000$ hours at the 5% significance level.

A sample of size $n = 20 \Rightarrow \bar{X} = 1014$ hours.

Test Statistic : $Z = \dfrac{\bar{X} - 1000}{25/\sqrt{20}}$

Reject Ho if $Z > Z_{0.05} = 1.645$

Observed test statistic $Z = \dfrac{1014 - 1000}{25/\sqrt{20}} = 2.5$

Since $2.5 > 1.645$ we reject Ho and conclude that there is strong evidence that the mean life of a light bulb exceeds 1000 hours.

(ii) p-value $= P(Z > 2.5) = 0.0062$

(iii) The smallest level of significance at which we can reject Ho is the p-value $= 0.0062$.

3. X : impact strength ; $X \sim N(\mu, \sigma^2)$

A sample of size 20 gave $\bar{X} = 1.25$ ft-lb/in
$s = 0.25$ ft-lb/in

Test at the 1% significance level

$H_0: \mu = 1$ ft-lb/in vs $H_a: \mu > 1$ ft-lb/in

Test statistic $t = \dfrac{\bar{X} - 1}{s/\sqrt{n}}$

Reject H_0 if $t > t_{0.01,19} = 2.539$

Observed test statistic $t = \dfrac{1.25 - 1}{0.25/\sqrt{20}} = 4.47$

Since $t > 2.539$ we reject H_0 and conclude that mean impact strength is greater than 1 ft-lb/in

(ii) p-value $= P(t_{19} > 4.47)$

< 0.0005

4. X: thickness of bottle ; $X \sim N(\mu, \sigma^2)$ σ^2 unknown

A sample of size $n = 40 \Rightarrow \bar{X} = 4.05$ mm

$s = 0.08$ mm

At the 5% significance level test

$H_0: \mu = 4.0$ mm vs $H_a: \mu > 4.0$ mm

Since n is large, the test statistic is

$$Z = \frac{\bar{X} - 4}{s/\sqrt{n}}$$

and we reject H_0 if $Z > 1.645$

Observed test statistic $Z = \dfrac{4.05 - 4}{0.08/\sqrt{40}} = 3.95$

Since $3.95 > 1.645$ we reject H_0 and conclude that the

mean thickness of the bottle exceeds 4 mm.

(ii) p-value $= P(Z > 3.95) \approx 0$

5. X_1 : deflection temperature of pipe type 1
X_2 : " " " " " 2

sample from population 1 of size $n_1 = 15$ gave

$\overline{X}_1 = 196.4$; $s_1 = 10.48$

sample of size $n_2 = 15$ from population 2 gave

$\overline{X}_2 = 192.07$; $s_2 = 9.44$

Test at the 5 % significance level

$H_0: \mu_1 - \mu_2 = 0$ vs $H_a: \mu_1 - \mu_2 < 0$.

Test statistic $t = \dfrac{\bar{X}_1 - \bar{X}_2}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

$$s_p = \sqrt{\frac{14s_1^2 + 14s_2^2}{28}} = 9.97$$

Reject H_0 if $t < -t_{0.05, 28} = -1.701$

Observed test statistic $t = 1.19$
Since $1.19 > -1.701$; we do not reject H_0 and conclude that there is not enough evidence to support that mean deflection temperature of type 2 exceeds mean deflection temperature of type 1.

(ii) Assumptions needed in (i) are

1. 2 populations are normal

2. Variances are equal

3. The 2 samples are independent.