

1.

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$X = \#$ of daily requests for assistance.

$$i) P(X \geq 4) = 0.12 + 0.10 + 0.08 + 0.07 + 0.06 + 0.04 + 0.03 \\ = 0.50$$

$$\text{OR } P(X \geq 4) = 1 - P(X \leq 3) \\ = 1 - (0.06 + 0.14 + 0.16 + 0.14) \\ = 0.50$$

$$ii) P(4 \leq X \leq 6) = 0.12 + 0.10 + 0.08 = 0.30$$

$$iii) \text{ Mean number of requests } \mu = \sum_{x=0}^{10} x \cdot p(x) \\ = 3.97 \text{ requests.}$$

$\sigma^2 =$ Variance of number of requests

$$= \sum x^2 p(x) - \mu^2$$

$$= 22.85 - 15.76$$

$$= 5.11$$

\Rightarrow Standard deviation of number of requests $= \sqrt{5.11} = 2.26$
(requests)

$$iv) P(\text{city ask for assistance}) = P(X > 3.97) \\ = P(X \geq 4) \\ = 0.50$$

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2/4

$X = \#$ of automobiles with high emissions

(i) $P(X=10) = (0.6)^{10}$

(ii) $P(X=6) = \binom{10}{6} (0.6)^6 (0.4)^4$

(iii) $P(X \geq 6) = \sum_{x=6}^{10} \binom{10}{x} (0.6)^x (0.4)^{10-x}$

(iv) $P(X \leq 3) = \sum_{x=0}^3 \binom{10}{x} (0.6)^x (0.4)^{10-x}$

(v) Assumptions: (i) each automobile has probability 0.6 of having high emissions

(ii) The automobiles are independent of one another in terms of having high emissions

(i) & (ii) $\implies X \sim B(10, 0.6)$

3. This is an example of the geometric random variable.
 $X = \#$ of trials until the 1st gear wheel fails.

$P(X=5) = (0.9)^4 (0.1)$

4.

A lot has 40 widgets; 6 of which are defective.

A sample of size $n=5$ is chosen.

$X = \#$ of defective widgets in the sample.

$$P(X \geq 2) = 1 - P(X \leq 1)$$

$$= 1 - [P(X=0) + P(X=1)]$$

$$= 1 - \left[\frac{\binom{34}{5}}{\binom{40}{5}} + \frac{\binom{6}{1}\binom{34}{4}}{\binom{40}{5}} \right]$$

5.

$X = \#$ of defective fuses in a sample of 6.

$$X \sim B(6, 0.08)$$

$$P(X < 2) = P(X \leq 1) = \sum_{x=0}^1 \binom{6}{x} (0.08)^x (0.92)^{6-x}$$

6.

If the selection is random, then

$$P(3 females are chosen) = \frac{\binom{3}{3}\binom{7}{1}}{\binom{10}{4}} = 0.033$$

which is very low.

Thus it is hard to believe that the personnel director's claim. He/she is probably biased.

7. Rate of accidents $\lambda = 0.3/\text{hr}$.
 $X = \#$ of accidents.

$$P(X > 4) = \sum_{x=5}^{\infty} \frac{e^{-(0.3)(10)} (3)^x}{x!}$$
$$= 1 - \sum_{x=0}^4 \frac{e^{-3} 3^x}{x!}$$