# Confidence Interval Estimation



1. X: life of a light bulb; $X \sim N(\mu, \sigma^2)$, $\sigma = 40$ hours

A sample of size $n = 30 \Rightarrow \bar{X} = 780$ hours.

a) a 96% confidence interval estimate for $\mu$ is

$$\bar{X} \pm Z_{0.02} \frac{\sigma}{\sqrt{n}} \quad ; \quad Z_{0.02} = 2.055$$

$\Leftrightarrow \quad 765 \le \mu \le 795$ hours.

b) The error in estimating $\mu$ is $E = 10$ hours;

then with 96% confidence $n = \dfrac{(2.055)^2 (40)^2}{(10)^2}$

$= 67.56$

Thus sample size needed is 68 light bulbs.

2. X: height of a college student ; $X \sim N(\mu, \sigma^2)$.

A sample of $n = 50$ students $\Longrightarrow$ $\bar{X} = 174.5$ cms.
$s = 6.9$ cms.

a) a 98% confidence interval estimate for $\mu$ is

$$\bar{X} \pm Z_{0.01} \frac{s}{\sqrt{n}} \qquad \text{Since } n \text{ is large.}$$

$\Leftrightarrow \quad 171.23 \le \mu \le 176.77$ cms.

(b) with 98% confidence we can assert that the error is at most 2.27 $\left(Z_{0.01} \frac{s}{\sqrt{n}}\right)$

2 $X_1$: yield of process using catalyst 1 ; $X_1 \sim N(\mu_1, \sigma_1^2)$

$X_2$: yield of process using catalyst 2 ; $X_2 \sim N(\mu_2, \sigma_2^2)$

A sample of size $n_1 = 12$ from process ~~of~~ 1 $\Rightarrow$ $\bar{x}_1 = 85$, $s_1 = 4$

A sample of size $n_2 = 10$ from process 2 $\Rightarrow$ $\bar{X}_2 = 81$, $s_2 = 5$

a) Need a 90% confidence interval estimate for $\mu_1 - \mu_2$

Assuming normal populations with equal variances.

a 90% confidence interval estimate for $\mu_1 - \mu_2$ is

$$\bar{X}_1 - \bar{X}_2 \pm t_{0.05,20} \, s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$t_{0.05,20} = 1.725$$

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} = \frac{11(16) + 9(25)}{20} = 20.05$$

Thus a 90% confidence interval estimate for $\mu_1 - \mu_2$ is

$[0.7, 7.3]$ ; i.e $0.7 \le \mu_1 - \mu_2 \le 7.3$

b) Assumptions : (i) Populations are normal
(ii) Variances are equal
(iii) 2 samples are independent.

(c) Interpretation of the interval:

The difference in the means yields of the two processes is between 0.7 and 7.3 with 90 % confidence. More precisely, since the interval is positive we can say that the mean yield of process 1 is greater than the mean yield of process 2 by an amount from 0.7 to 7.3 with 90 % confidence.

4. A sample of size $n = 1000$ produced $X = 228$ homes heated by oil

$\Longrightarrow$ Sample proportion of homes heated by oil; $\hat{p} = 0.228$

(a) a 99% confidence interval estimate for $p$, the true proportion of homes heated by oil is

$$\hat{p} \pm Z_{0.005}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \quad ; \quad Z_{0.005} = 2.55$$

$$\Longleftrightarrow \qquad 0.194 \leq p \leq 0.262$$

(b) $$n = \frac{(2.55)^2(0.228)(0.772)}{(0.05)^2} = 457.8$$

Thus sample size needed is only 458 homes.

5. A sample of $n = 100$ items gives $x = 8$ defectives.

$\therefore$ estimated proportion of defectives is $\hat{p} = \frac{8}{100} = 0.08$

a 98% confidence interval estimate for the true proportion of defectives $p$ is

$$\hat{p} \pm Z_{0.01}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\Leftrightarrow \quad 0.02 \leq p \leq 0.14$$

With no prior estimate of $p$; $n = \frac{(Z_{0.01})^2}{4E^2}$

$$n = \frac{(2.33)^2}{4(0.05)^2} = 542.89$$

Number of items to be sampled is 543 items.