Math101

Quiz#1

Code 001

Key

Name:

ID No:

Serial No:

1- The displacement (in meter) of a particle moving in a straight line is given by $s(t) = 3t^2 - 4t + 1$, where t is measured (in seconds).

a) Find the average speed over the time interval [0, 5].

average speed $= \dfrac{s(5) - s(0)}{5 - 0}$ (1)

$= \dfrac{56 - 1}{5}$

$= \dfrac{55}{5} = 11$ m/sec (1)

$s(5) = 3 \cdot 25 - 20 + 1$

$s(5) = 56$

$s(0) = 1$

2

b) Use limits to find the instantaneous speed of the particle when t=3.

instantaneous speed $= \lim\limits_{h \to 0} \left(\dfrac{s(3+h) - s(3)}{h} \right)$ (1)

$= \lim\limits_{h \to 0} \dfrac{3(3+h)^2 - 4(3+h) + 1 - [3(3)^2 - 4(3) + 1]}{h}$ (1)

$= \lim\limits_{h \to 0} \dfrac{\cancel{3(3)^2} + \boxed{3(2 \cdot 3h)} + \boxed{3h^2} - 4(3) \boxed{- 4h} + \cancel{1} - \cancel{3(3)^2} + 4(3) - \cancel{1}}{h} = \lim\limits_{h \to 0} \dfrac{14h + 3h^2}{h}$

$= \lim\limits_{h \to 0} (14 + 3h) = 14$ m/sec (1)

3

2- Evaluate the limit, if it exists

$\lim_{x \to 1/2} \left(\frac{2}{2x-1} - \frac{3}{2x^2+x-1} \right)$.

We cannot substitute $\boxed{x = \frac{1}{2}}$. 1

$\lim\limits_{x \to 1/2} \left(\dfrac{2}{2x-1} - \dfrac{3}{2x^2+x-1} \right) = \lim\limits_{x \to 1/2} \left(\dfrac{2}{2x-1} - \dfrac{3}{(2x-1)(x+1)} \right)$

1

$= \lim\limits_{x \to 1/2} \dfrac{2x+2-3}{(2x-1)(x+1)} = \lim\limits_{x \to 1/2} \dfrac{2x-1}{(2x-1)(x+1)}$ 1

1 $= \lim\limits_{x \to 1/2} \left(\dfrac{1}{x+1} \right) = \dfrac{1}{\frac{3}{2}} = \dfrac{2}{3}$. 1

5

3- Given that $\lim_{x\to 2}\left(3x-\frac{2}{5}\right)=\frac{28}{5}$ and $\epsilon = 0.009$. Find δ (the largest possible) that satisfies the condition given in the $\epsilon-\delta$ definition of a limit.

Need to find δ such that

$0<|x-2|<\delta \Rightarrow \left|\left(3x-\frac{2}{5}\right)-\frac{28}{5}\right|<\varepsilon.$ 2

But $\left|\left(3x-\frac{2}{5}\right)-\frac{28}{5}\right| = |3x-6| = 3|x-2| < \varepsilon = 0.009$ 1

5

So, $0<|x-2|<\delta \Rightarrow |x-2|<\frac{\varepsilon}{3}=\frac{0.009}{3}=0.003$ 1

Thus the largest possible value of δ is

$\boxed{\delta = 0.003}$. 1

4- Use Sandwich Theorem, to find

$\lim_{x\to 0}\left(2\pi - x + 5x^2\cdot\sin\frac{\pi}{x}\right).$

We know that $\boxed{-1 \le \sin\left(\frac{\pi}{x}\right) \le 1,}$ 1

then $2\pi - x - 5x^2 \le 2\pi - x + 5x^2\sin\frac{\pi}{x} \le 2\pi - x + 5x^2$

1

5

$\lim_{x\to 0}(2\pi - x - 5x^2) = 2\pi$ and $\lim_{x\to 0}(2\pi - x + 5x^2) = 2\pi$

2

Then by the Sandwich Theorem,

$\lim_{x\to 0}\left(2\pi - x + 5x^2\sin\frac{\pi}{x}\right) = 2\pi.$ 1