Math102 Name:

Quiz#1 ID No:

Code 001 Serial No:

1- If $y(x) = \int_{\sqrt{x}}^{\sqrt[4]{x}} \sin(t^2)\,dt$, find $\frac{dy}{dx}$

$$\boxed{\frac{d}{dx}\left[\int_{h(x)}^{g(x)} f(t)\,dt\right] = f(g(x))\,g'(x) - f(h(x))\,h'(x).}$$ $\frac{1}{2}$

$$\frac{dy}{dx} = \sin(\sqrt[4]{x})^2 \cdot (\sqrt[4]{x})' - \sin(\sqrt{x})^2 \cdot (\sqrt{x})' \quad 1$$

$$= \sin(\sqrt{x}) \cdot \frac{1}{4}\frac{1}{\sqrt[4]{x^3}} - \sin x \cdot \frac{1}{2\sqrt{x}} \quad 1$$

2- Evaluate $\int \frac{1}{\sqrt{x}e^{-\sqrt{x}}} \sec(e^{\sqrt{x}}+1)dx$

Let $\boxed{u = e^{\sqrt{x}}+1, \quad du = e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}\,dx}$ 1

$$I = 2\int \sec u\,du = 2\int \sec u \cdot \left(\frac{\sec u + \tan u}{\sec u + \tan u}\right) du$$

$$= 2\int \sec^2 u + \sec u \tan u \quad \frac{du}{\sec u + \tan u}$$

Let $\boxed{z = \sec u + \tan u, \quad dz = (\sec u \tan u + \sec^2 u)\,du}$ 1

$$I = 2\int \frac{dz}{z} = 2\ln|z|$$

$$= 2\ln|\sec u + \tan u| = \cancel{2\ln|\sec(e^{\sqrt{x}}+1)|}$$

$$= 2\ln\left|\sec(e^{\sqrt{x}}+1) + \tan(e^{\sqrt{x}}+1)\right| + C \quad \frac{1}{2}$$

3- If $y(x) = \int_{\sqrt{x}}^{0} \sin(t^2)\,dt$, find $\frac{dy}{dx}$

$= -\sin(\sqrt{x})^2 \cdot (\sqrt{x})'$ 1

$= -\sin(x) \cdot \left(\frac{1}{2\sqrt{x}}\right)$ 1

$= -\frac{\sin(x)}{2\sqrt{x}}$ ~~sin~~

$\frac{d}{dx}\left[\int_a^{g(x)} f(t)\,dt\right]$

$= f(g(x)) \cdot g'(x)$

$\frac{1}{2}$

4- Evaluate $\int \frac{1}{\sqrt{x}e^{-\sqrt{x}}} \csc(e^{\sqrt{x}} + 1)dx$

Let $\boxed{u = (e^{\sqrt{x}} + 1), \quad du = e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}\,dx \quad 2du = \frac{1}{\sqrt{x}\,e^{-\sqrt{x}}}dx}$ 1

$I = 2\int \csc u\,du = 2\int \frac{\csc u\,(\cot u + \csc u)}{(\cot u + \csc u)}\,du$

Let $\boxed{z = \cot u + \csc u, \quad dz = (-\csc^2 u - \csc u \cot u)\,du}$ 1

$I = -2\int \frac{dz}{z} = -2\ln|z|$

$= -2\ln|\cot u + \csc u|$

$= -2\ln\left|\cot(e^{\sqrt{x}}+1) + \csc(e^{\sqrt{x}}+1)\right| + C$ $\frac{1}{2}$

Math102 Name:

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1. Find the value of the following limit:

$$\lim_{n\to\infty}\left\{\sum_{i=1}^{n}\left[\boxed{\left(\frac{\pi}{n}\right)}\left(\cos\frac{i\pi}{2n}\right)^2\right]\right\} \text{ on } \left[0,\frac{\pi}{2}\right].$$

we need to divide by 2.

$$=2\int_0^{\frac{\pi}{2}}\cos^2 x\,dx \quad 1$$

$$=2\int_0^{\frac{\pi}{2}}\left(\frac{1+\cos 2x}{2}\right)dx \quad 1$$

$$=\left[x+\frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{2}}$$

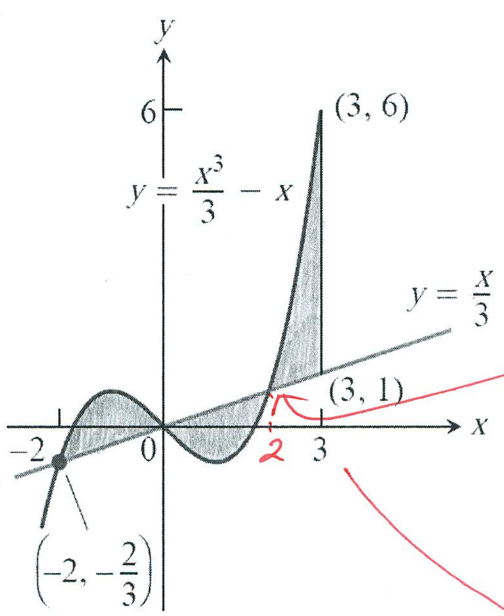
$$=\left[\left(\frac{\pi}{2}+\frac{1}{2}\sin\pi\right)-\left(0+\frac{1}{2}\sin 0\right)\right]$$

$$=\frac{\pi}{2}. \quad 1/2$$

- $\Delta x=\frac{b-a}{n}=\frac{\frac{\pi}{2}-0}{n}=\frac{\pi}{2n}$
- $f(x)=\cos^2 x$
- $\boxed{\cos^2 x=\frac{1+\cos 2x}{2}}$

$\int(\cos 2x)dx=\frac{1}{2}\int(\cos 2x)2\,dx=\frac{1}{2}\sin 2x+c.$

2. Set up the integration formula to find the area of the shaded region below:



$$A=\int_{-2}^{0}\left[\left(\frac{x^3}{3}-x\right)-\frac{x}{3}\right]dx \quad 1/2$$

$$+\int_{0}^{2}\left[\frac{x}{3}-\left(\frac{x^3}{3}-x\right)\right]dx \quad 1/2$$

$$+\int_{2}^{3}\left[\left(\frac{x^3}{3}-x\right)-\frac{x}{3}\right]dx \quad 1/2$$

intersection Pt:

$\frac{x}{3}=\frac{x^3}{3}-x$

$\frac{x^3}{3}-\frac{4x}{3}=0$

$x(x^2-4)=0$

$x=0,\ \pm 2$

$x=2$

1

$$\int \csc u \, \frac{\csc u + \cot u}{\csc u + \cot u} \, d$$

$$= \int \frac{\csc^2 u + \csc u \cot u}{\csc u + \cot u} \, du$$

$z = \csc u + \cot u$

$dz = -\csc u \cot u - \csc^2 u$

$$= -\int \frac{dz}{z}$$

$$= -\ln|z| .$$

3. Find the value of the following limit:

$$\lim_{n\to\infty}\left\{\sum_{i=1}^{n}\left[\left(\frac{\pi}{n}\right)\left(\sin\frac{i\pi}{2n}\right)^2\right]\right\} \text{ on } \left[0,\frac{\pi}{2}\right].$$

$$= 2\int_0^{\frac{\pi}{2}} \sin^2 x \, dx \quad 1$$

$$= \cancel{2}\int_0^{\frac{\pi}{2}} \left(\frac{1-\cos 2x}{\cancel{2}}\right) dx \quad 1$$

$$= \left[x - \frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{2}}$$

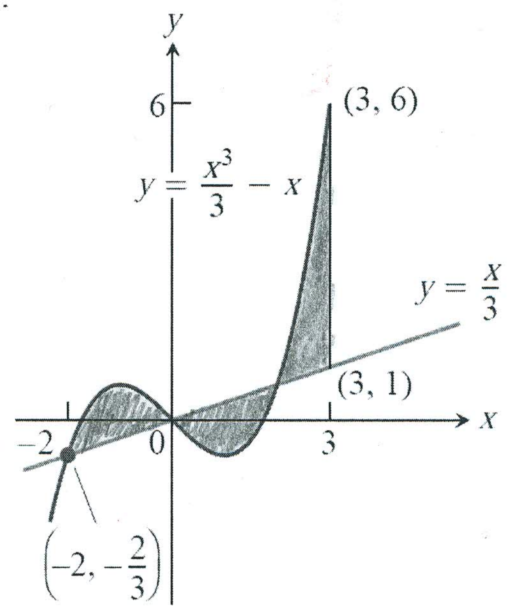
$$= \left[\left(\frac{\pi}{2} - \frac{1}{2}\sin\pi\right) - \left(0 - \frac{1}{2}\sin 0\right)\right] = \frac{\pi}{2} - \frac{1}{2}$$

- $\Delta x = \dfrac{\frac{\pi}{2} - 0}{n} = \dfrac{\pi}{2n}$
- $f(c_i) = \sin^2\left(\dfrac{i\pi}{2n}\right) = \sin^2(\Delta x \, i)$

$\Rightarrow \boxed{f(x) = \sin^2 x}$.

- $\boxed{\sin^2 x = \dfrac{1-\cos 2x}{2}}$

4. Set up the integration formula to find the area of the shaded region below:



See other Codes