31. $\int_{-1}^{4} \frac{dx}{\sqrt{|x|}} = \lim_{b \to 0^-} \int_{-1}^{b} \frac{dx}{\sqrt{-x}} + \lim_{c \to 0^+} \int_{c}^{4} \frac{dx}{\sqrt{x}} = \lim_{b \to 0^-} \left[-2\sqrt{-x}\right]_{-1}^{b} + \lim_{c \to 0^+} \left[2\sqrt{x}\right]_{c}^{4}$

$= \lim_{b \to 0^-} \left(-2\sqrt{-b}\right) - \left(-2\sqrt{-(-1)}\right) + 2\sqrt{4} - \lim_{c \to 0^+} 2\sqrt{c} = 0 + 2 + 2 \cdot 2 - 0 = 6$

32. $\int_{0}^{2} \frac{dx}{\sqrt{|x-1|}} = \int_{0}^{1} \frac{dx}{\sqrt{1-x}} + \int_{1}^{2} \frac{dx}{\sqrt{x-1}} = \lim_{b \to 1^-} \left[-2\sqrt{1-x}\right]_{0}^{b} + \lim_{c \to 1^+} \left[2\sqrt{x-1}\right]_{c}^{2}$

$= \lim_{b \to 1^-} \left(-2\sqrt{1-b}\right) - \left(-2\sqrt{1-0}\right) + 2\sqrt{2-1} - \lim_{c \to 1^+} \left(2\sqrt{c-1}\right) = 0 + 2 + 2 - 0 = 4$

37. $\int_{0}^{\pi} \frac{\sin\theta \, d\theta}{\sqrt{\pi-\theta}}$; $[\pi - \theta = x] \to -\int_{\pi}^{0} \frac{\sin x \, dx}{\sqrt{x}} = \int_{0}^{\pi} \frac{\sin x \, dx}{\sqrt{x}}$. Since $0 \le \frac{\sin x}{\sqrt{x}} \le \frac{1}{\sqrt{x}}$ for all $0 \le x \le \pi$ and $\int_{0}^{\pi} \frac{dx}{\sqrt{x}}$ converges, $\int_{0}^{\pi} \frac{\sin x}{\sqrt{x}}\, dx$ converges by the Direct Comparison Test.

38. $\int_{-\pi/2}^{\pi/2} \frac{\cos\theta \, d\theta}{(\pi - 2\theta)^{1/3}}$; $\begin{bmatrix} x = \pi - 2\theta \\ \theta = \frac{\pi}{2} - \frac{x}{2} \\ d\theta = -\frac{dx}{2} \end{bmatrix} \to \int_{2\pi}^{0} \frac{-\cos\left(\frac{\pi}{2} - \frac{x}{2}\right) dx}{2x^{1/3}} = \int_{0}^{2\pi} \frac{\sin\left(\frac{x}{2}\right) dx}{2x^{1/3}}$. Since $0 \le \frac{\sin\frac{x}{2}}{2x^{1/3}} \le \frac{1}{2x^{1/3}}$ for all $0 \le x \le 2\pi$ a[nd] $\int_{0}^{2\pi} \frac{dx}{2x^{1/3}}$ converges, then $\int_{0}^{2\pi} \frac{\sin\frac{x}{2}\, dx}{2x^{1/3}}$ converges by the Direct Comparison Test.