Math101 Key Name:

Quiz#1 ID No:

Serial No:

1- The displacement (in meter) of a particle moving in a straight line is given by $s(t) = 3t^2 - 4t + 1$, where t is measured (in seconds).

a) Find the average velocity over the time interval [0, 3].

$$\text{Averag Velocity} = \frac{S(3) - S(0)}{3 - 0} =$$

$$= \frac{16 - 1}{3}$$

$$= \frac{15}{3}$$

b) Use limits to find the instantaneous velocity of the particle when t=2.

$$\text{Instantaneous Velocity } (t=2) = \lim_{h\to 0} \frac{s(2+h) - s(2)}{h}$$

$$= \lim_{h\to 0} \frac{3(2+h)^2 - 4(2+h) + 1 - 5}{h}$$

$$= \lim_{h\to 0} \frac{8h + 3h^2}{h} = \lim_{h\to 0} 8 + 3h$$

$$= 8 \quad m/s$$

2- Evaluate the limit, if it exists

$\lim_{x\to 1/2}\left(\frac{2}{2x-1} - \frac{3}{2x^2+x-1}\right)$.

We cannot Substitute at $\boxed{x=\frac{1}{2}}$.

$$\lim_{x\to 1/2}\left(\frac{2}{2x-1} - \frac{3}{2x^2+x-1}\right) = \lim_{x\to 1/2}\left(\frac{2}{2x-1} - \frac{3}{(2x-1)(x+1)}\right)$$

$$= \lim_{x\to 1/2} \frac{2x+2-3}{(2x-1)(x+1)} = \lim_{x\to 1/2} \frac{2x-1}{(2x-1)(x+1)}$$

$$= \lim_{x\to 1/2}\left(\frac{1}{x+1}\right) = \frac{1}{\frac{3}{2}} = \frac{2}{3}.$$

3- Given that $\lim_{x\to 2}\left(3x-\frac{2}{5}\right)=\frac{28}{5}$ and $\epsilon=0.009$. Find δ (the largest possible) that satisfies the condition given in the $\epsilon-\delta$ definition of a limit.

Need to find δ such that

$0<|x-2|<\delta \Rightarrow \left|\left(3x-\frac{2}{5}\right)-\frac{28}{5}\right|<\varepsilon.$

But $\left|\left(3x-\frac{2}{5}\right)-\frac{28}{5}\right| = |3x-6| = 3|x-2| < \varepsilon = 0.009$

So, $0<|x-2|<\delta \Rightarrow |x-2| < \frac{\varepsilon}{3} = \frac{0.009}{3} = 0.003.$

Thus the largest possible value of δ is

$\boxed{\delta = 0.003}.$

4- Use Sandwich Theorem, to find

$\lim_{x\to 0}\left(\pi+x+x^2\cdot\sin\frac{\pi}{x}\right).$

we know, $\boxed{-1 \le \sin\frac{\pi}{x} \le 1}$

$-x^2 \le x^2\sin\frac{\pi}{x} \le x^2 \quad , \quad x^2 \ge 0$

$\pi + x - x^2 \le \pi + x + x^2 \sin\frac{\pi}{x} \le \pi + x + x^2$

Since $\lim_{x\to 0}(\pi+x-x^2) = \lim_{x\to 0}(\pi+x+x^2) = \pi$

Then by Sandwich Theorem

$\lim_{x\to 0}\left(\pi+x+x^2\sin\frac{\pi}{x}\right) = \pi.$