Math101

Quiz#2

Code 002

Key

Name:

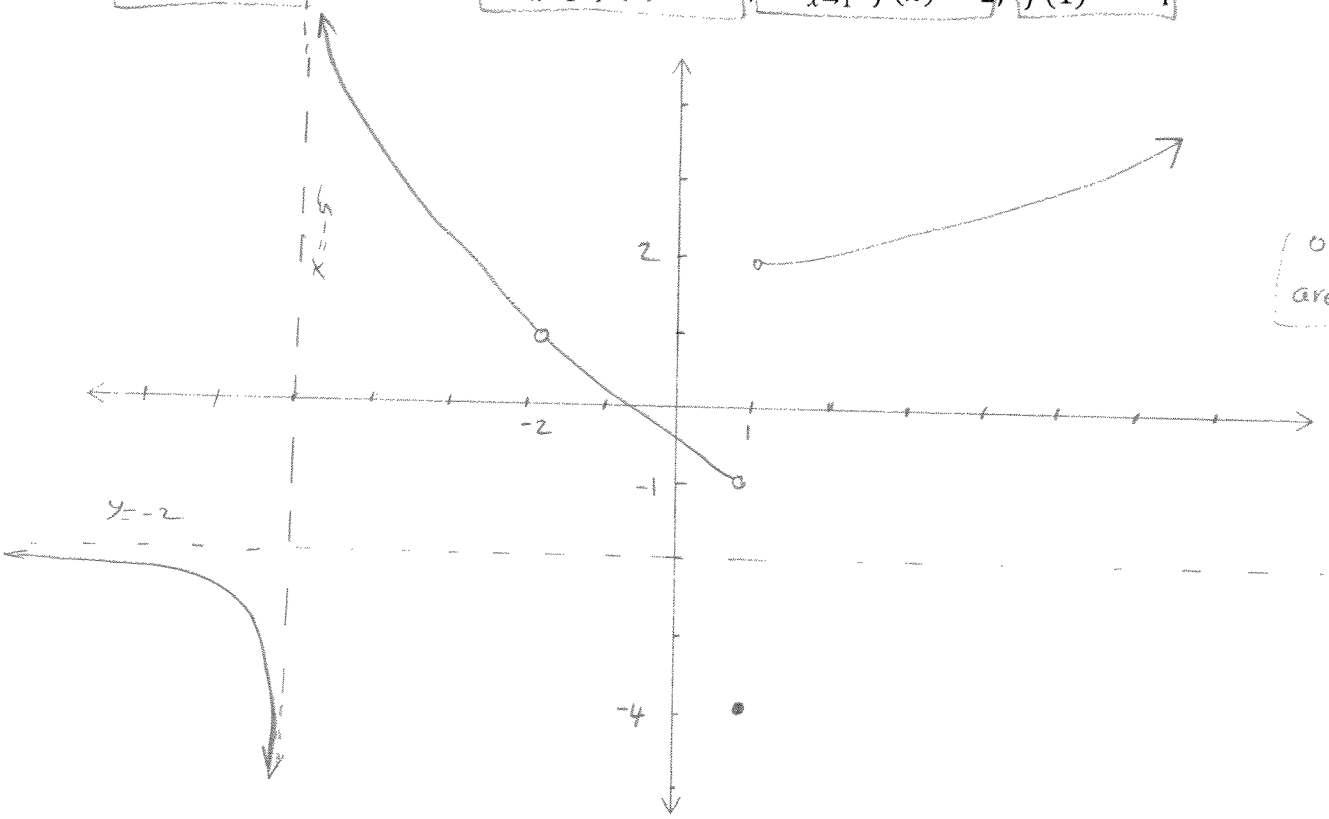
ID No:

Serial No:

1- *Sketch the graph of a function f that satisfies all of the following conditions:*

$\boxed{\lim_{x\to -5^-} f(x) = -\infty}$; $\boxed{\lim_{x\to -5^+} f(x) = \infty}$; $\boxed{\lim_{x\to -\infty} f(x) = -2}$; $\boxed{\lim_{x\to -2} f(x) = 1}$;

$\boxed{f(x) \text{ is undefined at } -2}$; $\boxed{\lim_{x\to 1^-} f(x) = -1}$; $\boxed{\lim_{x\to 1^+} f(x) = 2}$; $\boxed{f(1) = -4}$



other graphs are also possi

2- Let $f(x) = \begin{cases} \frac{6a}{x+1} & if\ x > 1 \\ 6 & if\ x = 1 \\ a^2 & if\ x < 1 \end{cases}$.

Find the value(s) of a so that $f(x)$ has a jump discontinuity.

To have a jump discontinuity, we need to have

$\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x) = f(1)$ OR $\lim_{x\to 1^+} f(x) \neq \lim_{x\to 1^-} f(x) = f(1)$

- $\lim_{x\to 1^-} f(x) = a^2$
- $\lim_{x\to 1^+} f(x) = \frac{6a}{2} = 3a$
- $f(1) = 6$

$3a = 6$ and $a^2 \neq 6$ OR $a^2 = 6$, i.e. $3a \neq 6$

$\boxed{a = 2}$

$\boxed{a = \pm\sqrt{6}}$

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3- *Use the Intermediate Value Theorem to show that the equation* $\cos x = \sqrt{x}$ *has at least one real root in the interval* $\left[0, \frac{\pi}{2}\right]$.

Let $\boxed{f(x) = \cos x - \sqrt{x}}$

Check $f(0) = 1 > 0$

and $f\left(\frac{\pi}{2}\right) = -\sqrt{\frac{\pi}{2}} < 0$.

So, ① $f(x)$ is continuous on $\left[0, \frac{\pi}{2}\right]$

② $f\left(\frac{\pi}{2}\right) < 0 < f(0)$,

therefore, by (IVT) $\exists\ c \in \left[0, \frac{\pi}{2}\right]$ s.t. $\boxed{f(c) = 0}$.

i.e. there exists at least one root in $\left[0, \frac{\pi}{2}\right]$.

4- *Evaluate the limit if it exists*

$$\lim_{x \to 1} \left(\frac{x^2 - |x-1| - 1}{|x-1|} \right)$$

. We need to check both side limits.

. $\lim_{x \to 1^-} \left(\frac{x^2 - |x-1| - 1}{|x-1|} \right) = \lim_{x \to 1^-} \left(\frac{x^2 - (1-x) - 1}{(1-x)} \right) = \lim_{x \to 1^-} \left(\frac{x^2 + x - 2}{1-x} \right)$

$= \lim_{x \to 1^-} \left(\frac{(x-1)(x+2)}{(1-x)} \right) = \lim_{x \to 1^-} -(x+2) = \boxed{-3}$

. $\lim_{x \to 1^+} \left(\frac{x^2 - |x-1| - 1}{|x-1|} \right) = \lim_{x \to 1^+} \left(\frac{x^2 - (x-1) - 1}{(x-1)} \right) = \lim_{x \to 1^+} \left(\frac{x^2 - x}{x-1} \right)$

$= \lim_{x \to 1^+} \left(\frac{x(x-1)}{(x-1)} \right) = \lim_{x \to 1^+} x = \boxed{1}$

Since $\lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x) \Rightarrow \lim_{x \to 1} f(x)$ DNE.

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$$\lim_{x\to 1}\left(\frac{x^2 - |x-1| - 1}{|x-1|}\right)$$

See code 002

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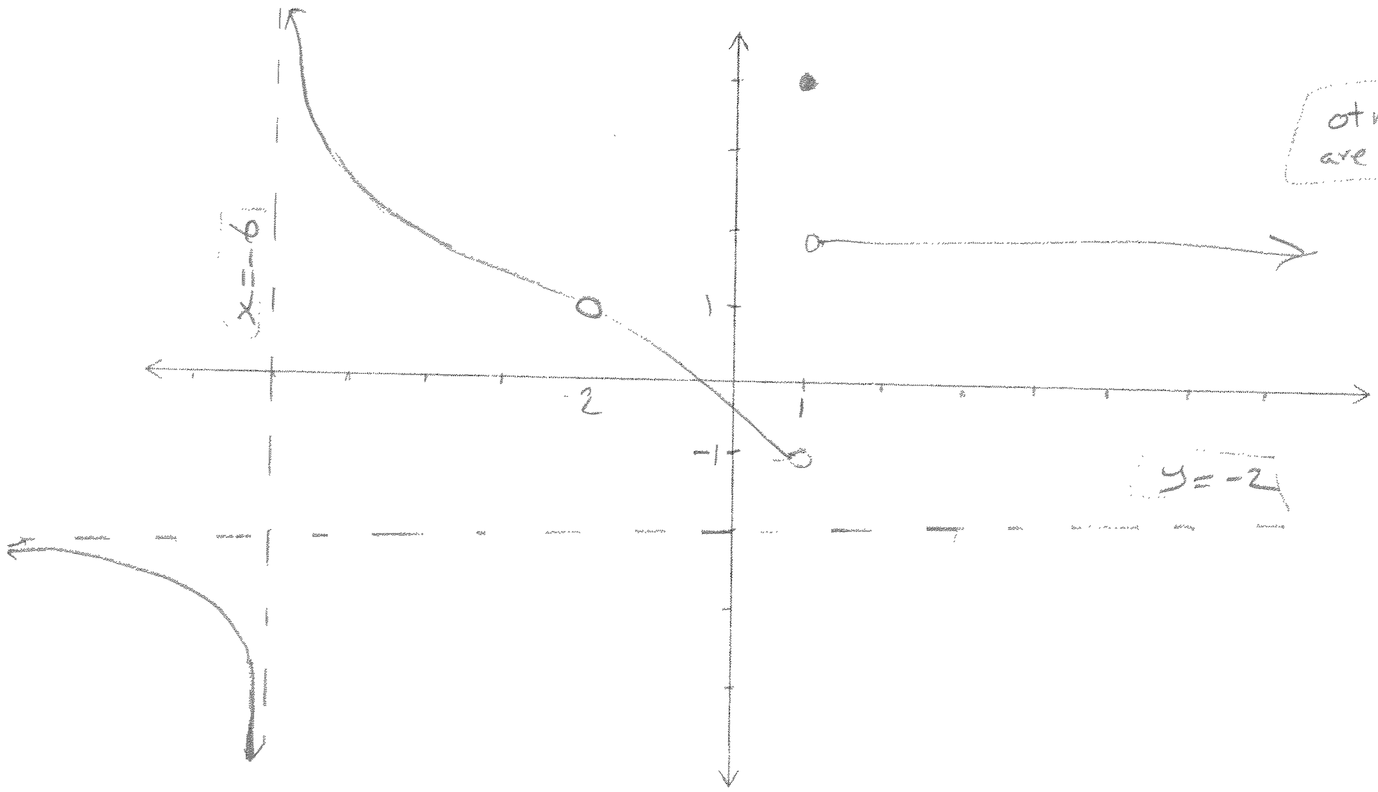
$f(x)$ is undefined at -2; $\lim_{x\to 1^-} f(x) = -1$; $\lim_{x\to 1^+} f(x) = 2$; $f(1) = 4$



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3- Let $f(x) = \begin{cases} \frac{6a}{x+1} & if\ x > 1 \\ 1 & if\ x = 1 \\ a^2 & if\ x < 1 \end{cases}$.

Find the value(s) of a so that $f(x)$ has a jump discontinuity.

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$$\left[\begin{array}{l} \lim_{x\to 1^-} f(x) = a^2 \\ \lim_{x\to 1^+} f(x) = \frac{6a}{2} = 3a \\ f(1) = 1 \end{array}\right\}$$

$3a = 1$ and $a^2 \neq 1$ $\Rightarrow \boxed{a = \frac{1}{3}}$ OR $a^2 = 1$ and $3a \neq 1$ $\boxed{a = \pm 1}$

4- *Use the Intermediate Value Theorem to show that the equation $\cos x = \sqrt{x}$ has at least one real root in the interval* $\left[0, \frac{\pi}{2}\right]$.

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