

Solve and then select the correct answer:

1. The sum of the absolute maximum and the absolute minimum values of the function

$$f(x) = 2 \cos x + 2 \cos^2 x, \quad \frac{\pi}{2} \leq x \leq 2\pi \quad \text{is}$$

- (a) 0
-  (b)  $\frac{7}{2}$
- (c) 1
- (d)  $\frac{5}{2}$
- (e) -1

2. Given that  $f(x) = \frac{x^2 + 1}{\sqrt{2x + 1}}$ , then the number of critical points of  $f$  is

- (a) 2
- (b) 4
-  (c) 1
- (d) 3
- (e) 0

3. If  $\cosh x = \frac{5}{3}$ , and  $x > 0$ , then  $\operatorname{csch} x =$

- (a)  $\frac{2}{5}$
-  (b)  $\frac{3}{4}$
- (c)  $\frac{4}{5}$
- (d)  $\frac{5}{3}$
- (e) None of the above.

4.

The function  $f(x) = 5x^3 - 3x^5$  has

- (a) 2 local extrema and one point of inflection
-  (b) 2 local extrema and 3 points of inflection
- (c) 3 critical points and no points of inflection
- (d) 2 local extrema and 2 points of inflection
- (e) 2 critical points and 3 points of inflection

5.

Let  $f(x) = \cos^2(x) + \sin(x)$ ,  $0 < x < \pi$ . Which one of the following statements is **TRUE**:

- (a)  $f$  is increasing on the interval  $\left(0, \frac{\pi}{3}\right) \cup \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$
-  (b)  $f$  is decreasing on the interval  $\left(\frac{\pi}{6}, \frac{\pi}{2}\right) \cup \left(\frac{5\pi}{6}, \pi\right)$
- (c)  $f$  is increasing on the interval  $\left(\frac{\pi}{3}, \frac{2\pi}{3}\right)$
- (d)  $f$  is decreasing on the interval  $\left(\frac{\pi}{6}, \frac{\pi}{2}\right) \cup \left(\frac{2\pi}{3}, \pi\right)$
- (e)  $f$  is decreasing on the interval  $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$

6.

If  $f(x) = \tanh(1 + e^{2x})$ , then  $\frac{df}{dx} \Big|_{x=0} =$

- (a)  $\frac{2}{e^2 - 2 + e^{-2}}$
-  (b)  $\frac{8}{e^4 + 2 + e^{-4}}$
- (c)  $\frac{4}{e^4 + 4 + e^{-4}}$
- (d)  $\frac{2}{e^4 + e^{-4}}$
- (e) None of the above.

7. The value(s) of  $c$  satisfying the conclusion of the Mean Value Theorem for the function  $f(x) = x + \frac{1}{x}$ , on the interval  $\left[\frac{1}{2}, 2\right]$  is (are)

(a)  $\frac{1}{2}$  and  $\frac{3}{2}$

→ (b) 1

(c)  $\frac{1}{4}$  and 1

(d) -1 and 1

(e) 1 and  $\frac{7}{4}$

8. The **number** of the critical points of  $f(x) = |x^3 - 4x|$  is

→ (a) 5

(b) 2

(c) 3

(d) 4

(e) 1

9. If  $M$  and  $m$  are the absolute maximum and absolute minimum values respectively of the function  $f(x) = x^3 - 3x^2 + 1$ ,  $-\frac{1}{2} \leq x \leq 4$ , then  $M - m =$

(a) 18

(b) 10

(c) 16

→ (d) 20

(e) 14

10. Let  $f(x) = x \ln x$ . Then the graph of  $f(x)$  is
- (a) increasing on  $(e, \infty)$  and decreasing on  $(0, e)$
  - (b) increasing on  $(0, \infty)$
  - (c) decreasing on  $(0, \infty)$
  - (d) increasing on  $\left(0, \frac{1}{e}\right)$  and decreasing on  $\left(\frac{1}{e}, \infty\right)$
  - (e) increasing on  $\left(\frac{1}{e}, \infty\right)$  and decreasing on  $\left(0, \frac{1}{e}\right)$

11. If  $(\alpha, \beta)$  is the point of inflection of the curve  $f(x) = \tan x - 4x$  for  $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ , then  $2\alpha - \beta + 2$
- (a) 0
  - (b) 2
  - (c)  $-\frac{\pi}{4}$
  - (d)  $\frac{\pi}{4}$
  - (e)  $1 - \pi$

12. Suppose that  $f'(x) \leq 1$  for  $1 \leq x \leq 4$ . Then the largest possible value of  $f(4) - f(1)$  is
- (a) 8
  - (b) 3
  - (c) 5
  - (d) 6
  - (e) 4