King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

**CODE 001** **Math 101** **CODE 001**

**Exam I (PART 1)**

**063**

**Tuesday 17/7/2007**

**Net Time Allowed: (For both parts) 70 minutes**

Name: ______________________________

ID: ____________________ Sec: ____________________.

**Check that this part has 7 questions.**

## Important Instructions:

1. All types of calculators, pagers or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The vertical and horizontal asymptotes of the graph of the function $f(x) = \dfrac{x^2-9}{x^2+2x-3}$ are

(a) $x = 1, x = -3, y = 1$

(b) $x = 1, y = -3$

(c) $x = 1, y = 1$

(d) $x = 1, y = -1$

(e) $x = -3, y = -1$

2. $\displaystyle\lim_{x\to -1^+} \frac{x^3-1}{x^2-1} =$

(a) $\infty$

(b) $\dfrac{1}{2}$

(c) $0$

(d) $\dfrac{3}{2}$

(e) $-\infty$

3. $\displaystyle\lim_{x \to 4^+} \frac{4 - x}{\sqrt{x} - 2} =$

(a) $\infty$

(b) $0$

(c) $-4$

(d) does not exist

(e) $4$

4. The value of the constant '$k$' that makes

$$g(x) = \begin{cases} x^3 + 2x + k + 3 & \text{if } x \leq 0 \\ \sqrt{x}\sin\dfrac{3}{x} & \text{if } x > 0 \end{cases}$$

continuous on $(-\infty, \infty)$ is

(a) $0$

(b) $3$

(c) $-1$

(d) $-3$

(e) $1$

5. The tangent line to the graph of a function $f(x)$ at $x = -1$ is $4x+y = 0$. Thus the value of the limit $\lim_{x\to -1} \dfrac{f(x) - f(-1)}{x+1}$ is

(a) $\infty$

(b) $-4$

(c) $1$

(d) $0$

(e) $4$

6. $\lim_{x\to 2} \arctan\left(\dfrac{x^2-4}{2\sqrt{3}x^2 - 4\sqrt{3}x}\right) =$

Note:

| $x$ | $0$ | $\pi/6$ | $\pi/4$ | $\pi/3$ |
|---|---|---|---|---|
| $\tan x$ | $0$ | $1/\sqrt{3}$ | $1$ | $\sqrt{3}$ |

(a) $0$

(b) $\dfrac{\pi}{3}$

(c) does not exist

(d) $\pi$

(e) $\dfrac{\pi}{6}$

7. $\lim_{x \to -\infty} \dfrac{\sqrt{3x^2+6}}{5-2x} =$

(a) $-\dfrac{\sqrt{3}}{2}$

(b) $0$

(c) $-\infty$

(d) $\dfrac{\sqrt{3}}{2}$

(e) $\infty$

# King Fahd University of Petroleum and Minerals
## Department of Mathematics and Statistics
## Math 101 –Calculus I
## Exam I
## Semester 071

November 6, 2007 Time: 7:00 - 8:45 PM

Section Number:______

ID Number: ____________.

Name: ______________________________________________

*Instructions* :

- (1) Write neatly and eligibly. You may lose points for messy work.
  (2) Show all your work. No credits for answers without justification.
  (3) All types of calculators and mobiles are not allowed.
  (4) Make sure that you have 7 different Problem. (7 pages +cover)

| Problem No | Student Grade | Maximum Points |
|---|---|---|
| 1 | | 15 |
| 2 | | 15 |
| 3 | | 14 |
| 4 | | 15 |
| 5 | | 15 |
| 6 | | 12 |
| 7 | | 14 |
| Total | | 100 |

1. Evaluate the limit if it exists:

(a) $\lim_{x\to 2} \dfrac{x^2-4}{x^3-8}$. (5 points)

(b) $\lim_{x\to 1^-} \dfrac{|x^2-3x+2|}{x^2-1}$. (5 points)

(c) $\lim_{x\to\infty} (\sqrt{x^2+x}-x)$. (5 points)

2. (a) Let $f(x) = 5x + 2$. Find the largest value of $\delta$ such that $|f(x) - 12| < 0.01$ whenever $-\delta < x - 2 < \delta$. (5 points)

(b) Show that there is a root of the equation $\sqrt[3]{x} = 1 - x$, between 0 and 1. (5 points)

(c) Find the limit if it exists:

$$\lim_{x \to 0} x^2 \cos\left(\frac{\pi}{x}\right).$$

(5 points)

3. (a) Find all values of $A$ and $B$ which will make the following function continuous:

$$f(x) = \begin{cases} x^2 - A & \text{if } x < 1, \\ A + Bx & \text{if } 1 \le x \le 2, \\ B - x^3 & \text{if } 2 < x. \end{cases}$$

(8 points)

(b) Find all horizontal and vertical asymptotes of $f(x) = \dfrac{\sqrt{4x^2+1}}{x+1}$, if any exists.

(6 points)

4. (a) A stone is thrown upward from ground level. Its height (in meters) after $t$ seconds is given by $s = 20t - 5t^2$. Find the following:

   (i) The average velocity of the stone during the first 3 seconds after being thrown. (2 points)

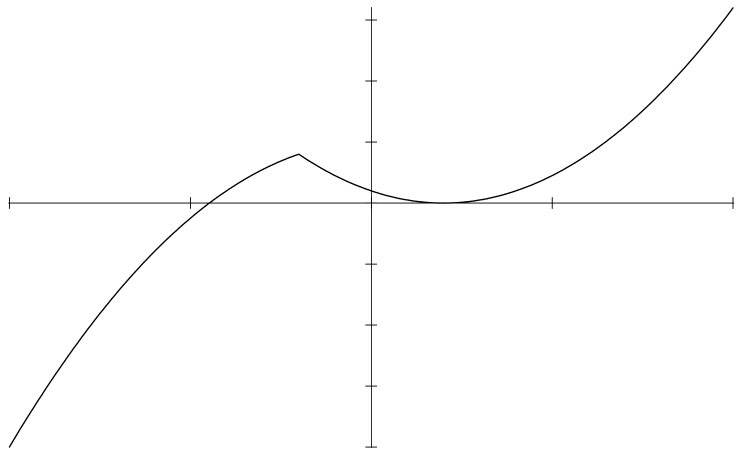
   (ii) The velocity of the stone after exactly 3 seconds of being thrown. (3 points)

   (iii) Find the maximum height reached by the stone. (3 points)

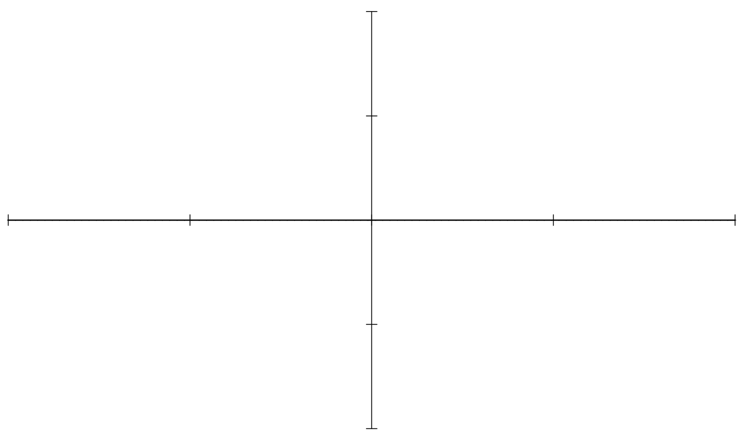
   (b) **Using the definition of the derivative**, find the slope of the tangent line of the curve $f(x) = \sqrt{2x+1}$ at $x = 4$. (7 points)

5. (a) Use the graph of $f(x)$, given below to sketch the graph of $f'(x)$. (7 points)

$f(x)$



$f'(x)$



(b) True or False? (8 points)

(i) Every differentiable function is continuous. **T** **F**

(ii) Every continuous function is differentiable. **T** **F**

(iii) The function $f(x) = |x^2 + 2|$ is continuous and differentiable. **T** **F**

(iv) The function $f(x) = \ln(4 - x^2)$ has vertical asymptotes. **T** **F**

6. (a) Let $f(x) = \dfrac{a+x}{b+x}$.

   (i) Find $f'(x)$ . (4 points)

   (ii) Find $a$ when $f'(0) = 4$ and $b = 2$. (2 points)

   (b) Determine the point $P(a, b)$ on the graph of $f(x) = e^x + x$, which has the property that the tangent line at $P$ is parallel to the line $y = 2x - 1$. (6 points)

7. (a) Find the derivative of $f(x) = \dfrac{\sqrt{x}\, e^x}{x+1}$ at $x = 1$. (6 points)

(b) Given $f(x) = \begin{cases} mx - 4, & \text{if } x < 2 \\ x^2 + m, & \text{if } x \geq 2 \end{cases}$,
find all values of $m$ that make $f$ differentiable at 2, if any exists. (8 points)

1. Evaluate the limit if it exists.

(a) $\lim\limits_{x\to 4} \dfrac{x^2-4x}{x^2-3x-4}$. (4 points)

(b) $\lim\limits_{x\to 1} \dfrac{\sqrt{x^2+1}-\sqrt{2}}{1-x}$. (6 points)

(c) $\lim\limits_{x\to 0^+} \dfrac{3}{x}\left(\dfrac{1}{4+x}-\dfrac{1}{4-x}\right)$. (6 points)

(d) $\lim\limits_{x\to 2^-} \left([x-1]-x^2\right)$, where $[\cdot]$ denotes the greatest integer function. (3 points)

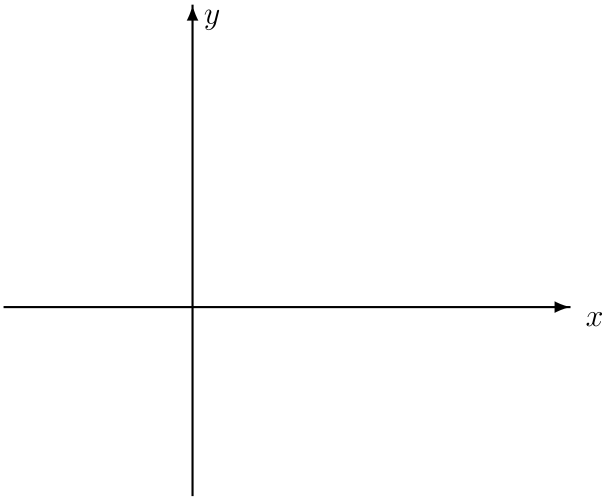
(e) $\displaystyle\lim_{x\to+\infty} \frac{\cos^2 x}{x^3}$. (6 points)

(f) $\displaystyle\lim_{x\to-\infty} (x + \sqrt{x^2 + 2x})$. (8 points)

2. Use the graph of $f(x) = \dfrac{1}{x}$ to find a number $\delta$ such that

$$\left|\frac{1}{x} - \frac{1}{3}\right| < \frac{1}{5} \quad \text{whenever} \quad |x - 3| < \delta.$$

(7 points)




3. Consider the function

$$f(x) = \begin{cases} x + 2 & \text{if } x < 0 \\ e^x & \text{if } 0 \le x \le 1 \\ e - \ln x & \text{if } x > 1 \end{cases}.$$

(a) Is $f$ continuous from the left at 0. Justify. (6 points)

(b) Is $f$ continuous at 1. Justify. (6 points)

4. Where is the function $f(x) = \dfrac{\sin\left(\dfrac{1}{x}\right)}{e^x - 2}$ continuous. (6 points)

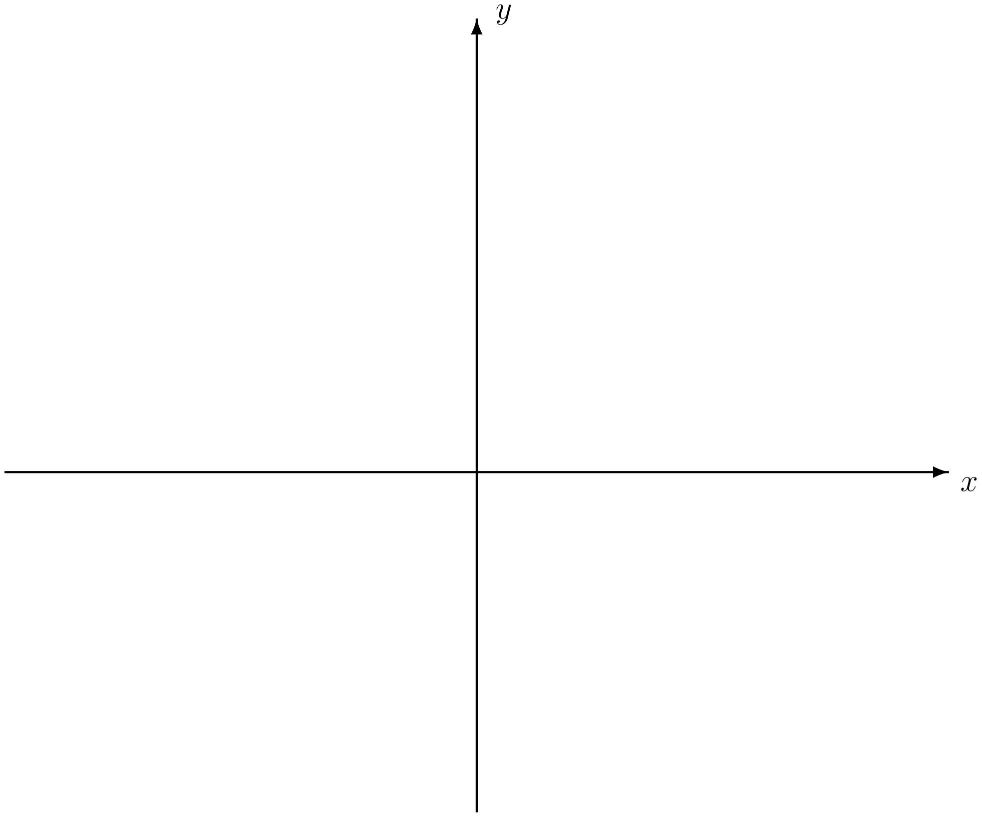
5. Show that the equation $e^x = -1 - 2x$ has a root in the interval $(-1, 0)$. (8 points)

6. Find the horizontal and vertical asymptotes of the graph of the function $f(x) = \dfrac{4x-1}{x^3-8x^2}$. Explain. (9 points)

7. Sketch the graph of a function $f$ that satisfies all of the given conditions:

$$f(0) = 0, f(2) = 1, \lim_{x\to 2^+} f(x) = 0, \lim_{x\to 2^-} f(x) = 2,$$
$$\lim_{x\to +\infty} f(x) = 4, \lim_{x\to -\infty} f(x) = -1, \lim_{x\to -2} f(x) = +\infty.$$

(8 points)



8. Find an equation of the tangent line to the curve $y = \dfrac{1}{x-2}$ at the point $\left(4, \dfrac{1}{2}\right)$.
   [**You must use limits**] (8 points)

9. The position function of a particle moving in a straight line is given by the equation of motion $s = t^3 - 2t$, where $t$ is measured in seconds and $s$ in meters.
   (a) Find the average velocity of the particle over the time interval $[1, 3]$. (3 points)

   (b) Use limits to find the instantaneous velocity of the particle when $t = 2$. (6 points)

**King Fahd University of Petroleum and Minerals**
**Department of Mathematics and Statistics**
**Math 101- Calculus I**
**Exam I**
**2007-2008 (073)**

**Tuesday, July 22, 2008** **Allowed Time: 2 hours**

**Name: ___________________________________**

**ID Number: ____________**

**Section Number:___________** **Serial Number:___________**

**Instructions:**

1. Write neatly and eligibly. You may lose points for messy work.
2. **Show all your work.** No points for answers without justification.
3. **Calculators and Mobiles are not allowed.**
4. Make sure that you have 8 different problems (6 pages + cover page)

| Problem No | Grade | Maximum Points |
|---|---|---|
| 1 | | 33 |
| 2 | | 7 |
| 3 | | 7 |
| 4 | | 8 |
| 5 | | 8 |
| 6 | | 13 |
| 7 | | 8 |
| 8 | | 16 |
| Total | | 100 |

**MATH 101, EXAM-I (Term 073)**

1. Evaluate the limit if it exists. Justify your answer

(a) $\lim\limits_{x\to 0^+} \dfrac{x-1}{x^2+2x}$. (4 pts.)

(b) $\lim\limits_{x\to 1} \dfrac{\sqrt{x}-x^2}{1-\sqrt{x}}$. (8 pts.)

(c) $\lim\limits_{x\to 0^-} x\sin\left(\dfrac{\sqrt{x+2}}{x}\right)$. (6 pts.)

(d) $\lim_{x \to 1} arc\sin\left(\frac{1-x}{1-x^2}\right)$. (4 pts.)

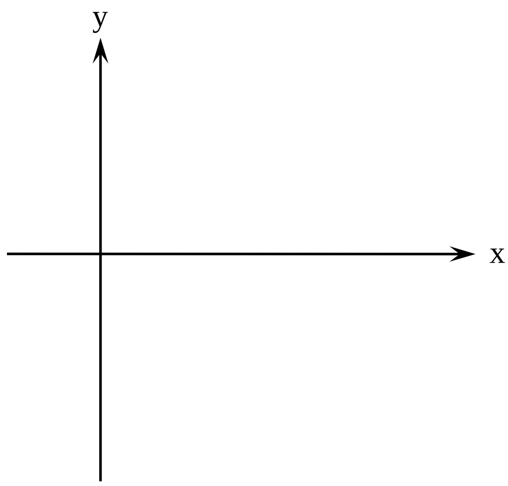
(e) $\lim_{x \to -\infty} \frac{x^3-2x+7}{-2x^2+x-3}$. (4 pts.)

(f) $\lim_{x \to +\infty} \left(\sqrt{9x^2+x}-3x\right)$. (7 pts.)

**MATH 101, EXAM-I (Term 073)**

2. Use the graph of $f(x)=\sqrt{x-1}$ to find a number $\delta$ such that (7 pts.)

$$\left|\sqrt{x-1}-1\right|<0.1 \text{ whenever } |x-2|<\delta.$$




3. Where is the function $f(x)=\dfrac{1}{1-e^{\frac{x+1}{x}}}$ continuous? (7 pts.)

**MATH 101, EXAM-I (Term 073)**

4. Find the constant $k$ that makes the function (8 pts.)

$$f(x) = \begin{cases} x^2 - k^2 & \text{if} \quad x \le 2 \\ kx + 5 & \text{if} \quad x > 2 \end{cases}$$

continuous on $(-\infty, +\infty)$.

5. Show that the equation $x \ln x = \sin x$ has a root in the interval $(1, e)$. (8 pts.)

6. (a) How many horizontal asymptotes can a function have? (6 pts.)
Illustrate your answer graphically.

(b) Does the graph of $f(x) = \ln\left(9 - x^2\right)$ have a vertical asymptote (7 pts.)

(i) at $x = 3$. Justify.

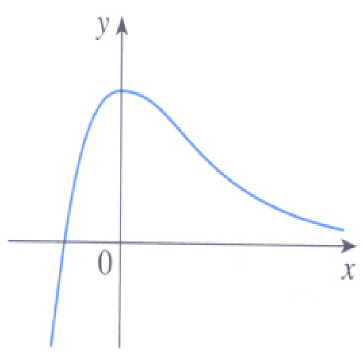
(ii) at $x = -1$. Justify.

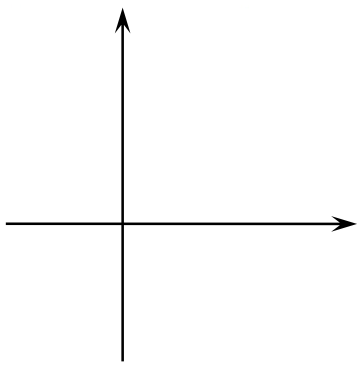
7. The position function of a particle moving in a straight line is given by the equation of motion $s(t) = \dfrac{1-t}{1+t}$, where $t$ is measured in seconds and $s$ in meters. Find the instantaneous velocity of the particle when $t = 1$. (8 pts.)

8. (a) **TRUE** or **FALSE**. Justify: If $f'(a)$ exists, then $\lim_{x\to a} f(x) = f(a)$. (4 pts.)

(b) Is $f(x) = x|x|$ differentiable at $x = 0$. Justify. (6 pts.)

(c) Graph the derivative of the function whose graph is given below. (6 pts.)



1. (10 points) Find each of the following limits of the function $f$ whose graph is given in the adjacent figure

(a) $\lim_{x\to -2^-} f(x) =$

(b) $\lim_{x\to -1} f(x) =$

(c) $\lim_{x\to 0^-} f(x) =$

(d) $\lim_{x\to 0^+} f(x) =$

(e) $\lim_{x\to 1} f(x) =$

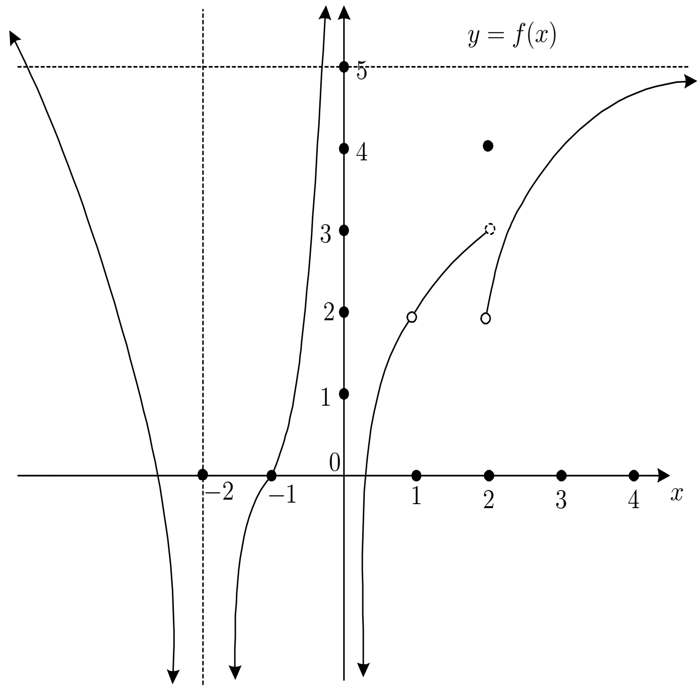
(f) $\lim_{x\to 2^-} f(x) =$

(g) $\lim_{x\to 2^+} f(x) =$

(h) $\lim_{x\to 2} f(x) =$

(i) $\lim_{x\to -\infty} f(x) =$

(j) $\lim_{x\to +\infty} f(x) =$




2. (7 points) Sketch the graph of an example of a function $f$ that satisfies the following conditions:

(a) $f'(-3) = f'(3) = 0,$

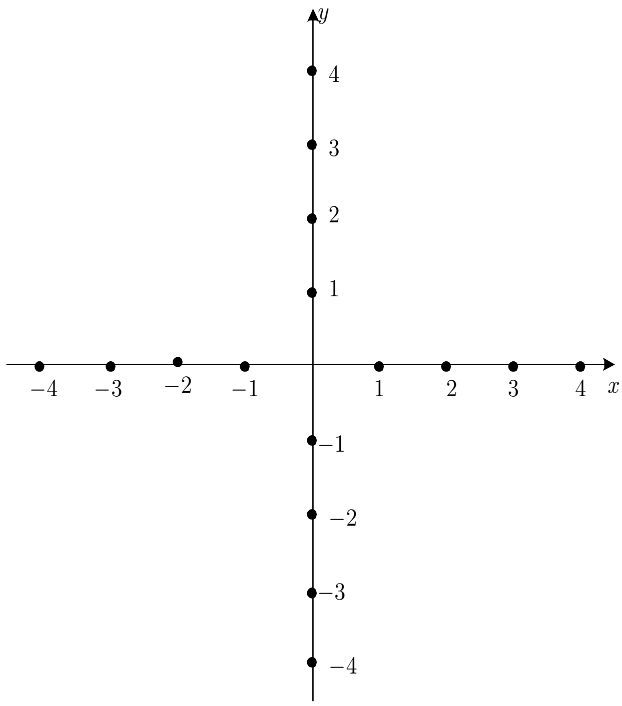
(b) $\lim_{x\to 0^-} f(x) = -1,$

(c) $\lim_{x\to 0^+} f(x) = 1,$

(d) $f(0)$ is undefined,

(e) $\lim_{x\to 2} f(x) = -1,$

(f) $f(2) = 1.$



3. Evaluate each of the following limits (show your steps).

(a) (3 points) $\displaystyle\lim_{x\to 2} \frac{x^2 - 3x + 2}{2 - x}$.

(b) (4 points) $\displaystyle\lim_{x\to +\infty} \frac{1 - x - 2x^3}{x^3 + 2x^2 + 1}$.

(c) (4 points) $\displaystyle\lim_{x\to -\infty} \frac{\sqrt{3x^2 + 7}}{4x - 11}$.

(d) (4 points) $\displaystyle\lim_{x\to \frac{1}{2}^-} \frac{12x^2 - 6x}{|2x - 1|}$.

4. (4 points) If $\lim_{x\to 2} f(x) = 7$ and $\lim_{x\to 2} g(x) = 3$, find $\displaystyle\lim_{x\to 2} \frac{\sqrt{x+f(x)}}{|x-2|-(g(x))^2}$. **Justify each step.**

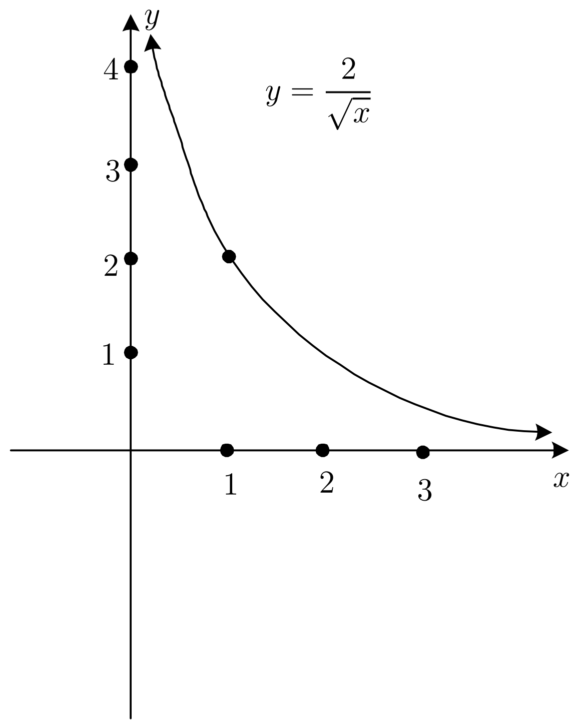
5. (10 points) Use the Squeeze Theorem to show that $\displaystyle\lim_{x\to 0} \sin x \cdot \cos\frac{1}{x} = 0$.

6. The displacement (in meters) of a particle moving in a straight line is given by the equation $S = 40 + 16t^2$, where $t$ is measured in seconds.

   (a) (3 points) Find the average velocity of the particle over the time interval with endpoints between $1$ and $1+h$.

   (b) (2 points) Use part (a) to find the instantaneous velocity of the particle when $t = 1$.

7. (9 points) Use the graph of $f(x) = \dfrac{2}{\sqrt{x}}$ to find the largest a number $\delta$ such that $|f(x) - 2| < \dfrac{1}{2}$ whenever $0 < |x-1| < \delta$. $\left(\text{Show your steps and write your answer in a rational form } \dfrac{p}{q}\right)$.



8. (8 points) Find an equation of the tangent line to the curve $f(x) = \dfrac{2}{x+3}$ at the point where $x = -1$. [You must use limits].

9. (9 points) If $[x]$ denotes the greatest integer less than or equal to $x$, find all values of $x$ for which the following function is continuous:

$$f(x) = \begin{cases} [x], & \text{if } -2 \leq x < 0 \\ x, & \text{if } 0 \leq x < 1 \\ 3x - 2, & \text{if } 1 \leq x \leq 2 \end{cases}$$

(Use limits to justify your answers).

10. (6 points) Determine whether the function

$$f(x) = \frac{\sqrt{2x+9} - \sqrt{x+9}}{2x}$$

has a removable discontinuity, a jump discontinuity, or an infinite discontinuity at $x = 0$.

11. (5 points) Use the Intermediate Value Theorm to show that there is a root of the equation $x^6 + x^4 - 1 = 0$ in the interval $[-1, 1]$.

12. (4 points) The limit $\lim_{x \to \frac{\pi}{2}} \dfrac{6(\sin x - 1)}{2x - \pi}$ represents the derivative of some function $f$ at some number $a$. State such an $f$ and $a$. (give a reason to your answer)

13. (8 points) Find the equations of all horizontal asymptotes to the graph of $f(x) = \tan^{-1}(e^{-2x} - 1)$. (Show your work)

1. (a) [**3 points**] Write the following statement as a limit:

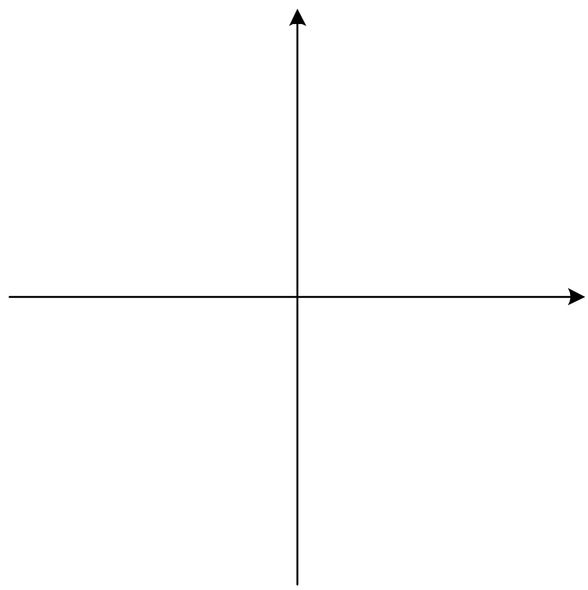
"$f(x)$ increases without bound as $x$ approaches $a$ from the left".

(b) [**4 points**] TRUE or FALSE: "If $f$ has a domain $[0, +\infty)$ and has no horizontal asymptote, then $\lim_{x \to +\infty} f(x) = +\infty$ or $\lim_{x \to +\infty} f(x) = -\infty$".

[If TRUE, state the reason. If FALSE, illustrate graphically].

(c) [**7 points**] Sketch the graph of a function $f$ that satisfies the following conditions:

i. $f(-1) = 3$
ii. $\lim_{x \to -1^-} f(x) = 4$
iii. $\lim_{x \to -1^+} f(x) = -\infty$
iv. $f(3)$ is undefined
v. $\lim_{x \to 3} f(x) = 2$
vi. $\lim_{x \to +\infty} f(x) = +\infty$
vii. $\lim_{x \to -\infty} f(x) = 0$



2. Find the limit if it exists.

   (a) [**6 points**] $\lim_{x \to -4} \dfrac{x^3 - 16x}{x + 4}$

   (b) [**6 points**] $\lim_{x \to 12} \dfrac{|12 - x|}{x - 12}$

   (c) [**6 points**] $\lim_{x \to 3} g(x)$, where $2x - 1 \leq g(x) \leq x^2 - 5x + 11$

   (d) [**6 points**] $\lim_{x \to 6^+} \tan^{-1}(\ln(x - 6))$

3. [**8 points**] Using the $\epsilon, \delta$ definition of limit, prove that $\lim_{x \to 1} \left( -1 + \frac{3}{2}\, x \right) = \frac{1}{2}$

4. [**8 points**] Let $f(x) = \begin{cases} \sqrt{x+2} & \text{if} \quad -2 \le x \le 2 \\ x^3 - 2x & \text{if} \quad x > 2\,. \end{cases}$ Is $f$ continuous at $x = 2$. If not, what kind of discontinuity does $f$ have at $x = 2$. Justify your answers.

5. [**6 points**] Where is the function $f(x) = \dfrac{1}{3 - \sqrt{x}}$ continuous?

6. [**8 points**] Show that the equation $e^{-x} = 2 - x$ has a root in the interval $(1, 2)$.

7. (a) [**8 points**] Find $\lim_{x \to +\infty} (\sqrt{x^2 + 1} - x)$.

   (b) [**8 points**] Find the horizontal asymptotes of $f(x) = e^{x - x^2}$.

8. [**8 points**] Find an equation of the tangent line to the curve $y = \dfrac{1}{x^2 - x}$ at the point $\left(2, \dfrac{1}{2}\right)$. [You must use limits]

9. The displacement (in meters) of a particle moving in a straight line is given by the equation $s(t) = 3t^2 - 4t + 1$, where $t$ is measured in seconds.

   (a) [**2 points**] Find the average velocity over the time interval $[0, 3]$.

   (b) [**6 points**] Use limits to find the instantaneous velocity when $t = 2$.

**King Fahd University of Petroleum and Minerals**
**Department of Mathematics and Statistics**

**MATH 101 EXAM I**
Summer Term (083)

**Time allowed: 120 Minutes**

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Name: ______________________________ ID#:__________

Instructor: __________ Section:__________ Serial#:__________

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- **Show All Your WORK**
- **WRITE Clear Steps**
- **Calculator and Mobiles are not allowed**

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| Q# | Marks | Maximum Marks |
|---|---|---|
| 1 | | 8 |
| 2 | | 10 |
| 3 | | 24 |
| 4 | | 10 |
| 5 | | 8 |
| 6 | | 8 |
| 7 | | 7 |
| 8 | | 7 |
| 9 | | 8 |
| 10 | | 10 |
| **Total** | | 100 |

**MATH 101 - EXAM (Term 083)**

1. **(8 - points)** Sketch the graph of an example of a function $f$ that satisfies the following conditions:

$\lim_{x\to-\infty} f(x) = 3$; $\lim_{x\to\infty} f(x) = 1$; $\lim_{x\to 1^-} f(x) = -\infty$;

$f'(-2) = 0$; $\lim_{x\to 1^+} f(x) = 2$; $f$ has a removable discontinuity at $x = -1$.

2. **(10 - points)** Use the Squeeze Theorem to show that

$$\lim_{x\to 0^+}\left(\sqrt{x}\; e^{\sin\left(\frac{\pi}{\sqrt{x}}\right)} + 1\right) = 1$$

**MATH 101 - EXAM (Term 083)**

3. **(24 points: 6 points each)** Evaluate the limit, if it exists

(3a) $\lim_{x \to 1} \frac{x^3 - 1}{\sqrt{2x+2} - 2}$

(3b) $\lim_{x \to 1^-} \frac{x^2 - |x-1| - 1}{|x-1|}$

(3c) $\lim_{x \to \frac{1}{2}} (x - [|2x|])$, where $[|\ \ |]$ denotes the greatest integer function.

**MATH 101 - EXAM (Term 083)**

(3d) $\lim_{x\to\infty} \ln\left(\dfrac{e^{x+2}-8}{e^x+16}\right)$

4. **(10 - points)** The displacement (in meters) of a particle moving in a straight line is given by the equation of motion $S(t) = \dfrac{3t-1}{t+2}$ where $t$ is measured in seconds. Use limits to find the instantaneous velocity at $t = 3$.

**MATH 101 - EXAM (Term 083)**

5. **(8 - points)** Use the Intermediate Value Theorem to show that the graphs of the functions $f(x) = \sqrt{x}$ and $g(x) = \cos x$ intersect on the interval $\left[0, \dfrac{\pi}{2}\right]$.

6. Given that $f(x) = (x-1)^{\frac{2}{3}}$ and $f'(x) = \frac{2}{3}(x-1)^{-\frac{1}{3}}$.

   (6a) **(3 - points)** Use limits to find, if any, the equation of the vertical tangent to the graph of $f$.

   (6b) **(5 - points)** Find the equation of the normal line to the graph of $f$ at $x = 9$.

**MATH 101 - EXAM (Term 083)**

7. **(7 - points)** Determine the intervals on which the function $f(x) = \dfrac{\ln(x) + \tan^{-1}(3x)}{x^2 - 4}$ is continuous.

8. **(7 - points)** Use limits to determine whether or not the following function is continuous at $x = 2$

$$f(x) = \begin{cases} \dfrac{10}{3x-1}, & \text{if } x < 2 \\ \sqrt{3x-2}, & \text{if } x \geq 2 \end{cases}$$

**MATH 101 - EXAM (Term 083)**

9. **(8 - points)** Given that $\lim_{x\to 2}\left(3x-\frac{2}{5}\right)=\frac{28}{5}$ and $\epsilon = 0.009$. Find the largest possible value of $\delta$ that satisfies the conditions given in the $\epsilon - \delta$ definition of a limit.

10. **(10 - points)** Use limits to find all vertical and horizontal asymptotes of the graph of

$$f(x)=\frac{6x}{\sqrt{2x^2-8}}$$

King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics
MATH 101 – Calculus I
EXAM I
2009-2010 (091)

**Monday, November 2, 2009** **Allowed Time: 2 Hours**

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**Name:** ______________________________

**ID Number:** ____________________ **Serial Number:** ____________________

**Section Number:** ______________ **Instructor's Name:** ____________________

**Instrunctions**:

1. Write neatly and legibly. You may lose points for messy work.
2. **Show all your work.** No points for answers without justification.
3. **Calculators and Mobiles are not allowed**.
4. Make sure that you have 10 different problems (6 pages + cover page).

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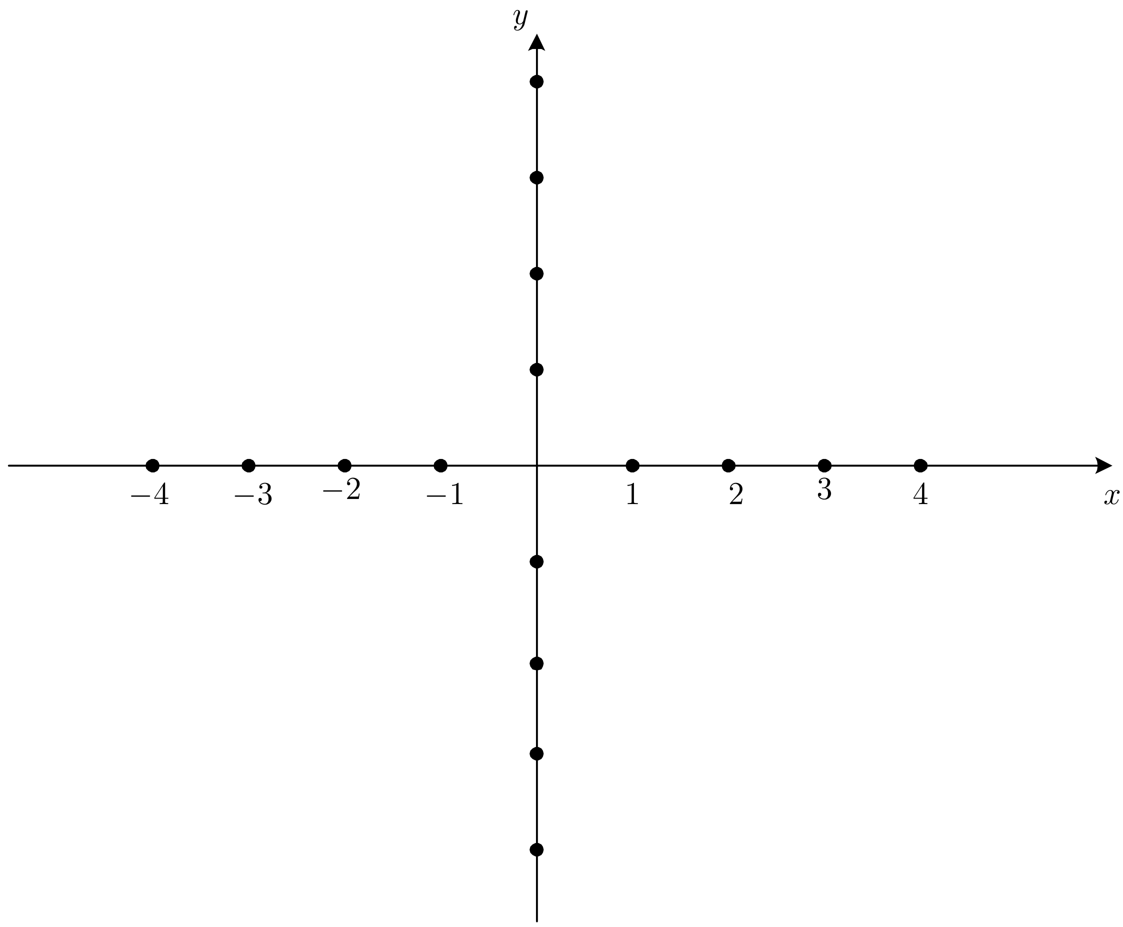
| Problem No. | Points | Maxiumum Points |
|---|---|---|
| 1 | | 12 |
| 2 | | 7 |
| 3(a,b,c) | | 18 |
| 4 | | 12 |
| 5 | | 6 |
| 6 | | 10 |
| 7 | | 7 |
| 8 | | 12 |
| 9 | | 12 |
| 10 | | 4 |
| Total: | | 100 |

1. (12-points) Sketch the graph of a function $f$ that satisfies all of the following conditions:

   $\lim_{x\to -4^-} f(x) = -\infty\,;$ $\quad \lim_{x\to -4^+} f(x) = \infty\,;$ $\quad \lim_{x\to -\infty} f(x) = 0;$

   $\lim_{x\to -2} f(x) = 1\,;$ $\quad f$ is undefined at $-2;$

   $\lim_{x\to 1^-} f(x) = -1\,;$ $\quad \lim_{x\to 1^+} f(x) = 2\,;$ $\quad f(1) = 1$




2. (7-points) If $x^3 - x + 4 \le x + f(x) \le 3x^2 + 1$ for all real numbers $x$, then find $\lim_{x\to 1} f(x)$. (Give reasons to your steps).

3. Evaluate the limit, if it exists:

   (a) (6-points) $\lim_{x \to 1/2} \left( \frac{2}{2x-1} - \frac{3}{2x^2+x-1} \right)$.

   (b) (6-points) Let $f(x) = \left[ \frac{1}{2}x + 1 \right]$ be the greatest integer less than or equal to $\frac{1}{2}x + 1$. Find each of the following limits:

   (i) $\lim_{x \to -2^-} f(x)$

   (ii) $\lim_{x \to -2^+} f(x)$

   (iii) $\lim_{x \to -2} f(x)$

   (c) (6-points) $\lim_{x \to 3^-} \frac{|x^2-9|}{x-3}$.

4. (12-points) Find the horizontal asymptotes of the graph of the function

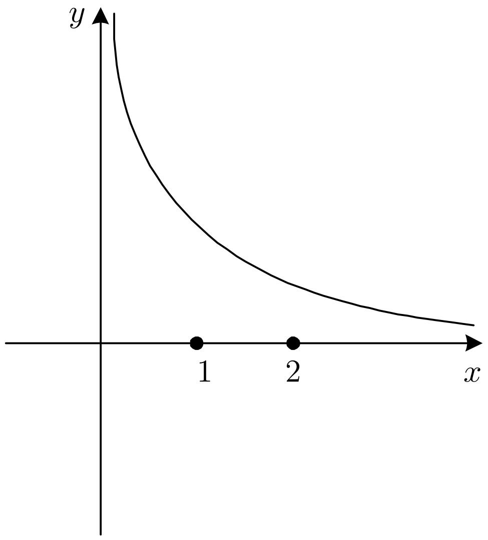
$$f(x) = \arctan \frac{\sqrt{9x^2+2}}{3x+7}.$$

5. (6-points) Let $f(x) = \dfrac{4-x^2}{2-x-x^2}$. Find the following limits (write the answer as a real number, $\infty$, or $-\infty$).

   (a) $\lim\limits_{x\to 1^-} f(x)$.

   (b) $\lim\limits_{x\to 1^+} f(x)$.

6. (10-points) Use the graph of $f(x) = \dfrac{1}{x}$ to find the largest number $\delta$ such that if $|x-1| < \delta$, then $|f(x) - 1| < 0.1$. (Show your work and write your answer in simplest rational form $\dfrac{p}{q}$).




7. (7-points) Use the Intermediate Value Theorem to show that there is a root of the equation $e^{-x^2} = x$ between 0 and 1.

8. (12-points) The displacement (in meters) of a particle moving in a straight line is given by $s = \dfrac{1}{\sqrt{5-t}}$ where $t$ is measured in seconds. Use limits to find the instantaneous velocity of the particle when $t = 1$.

9. (12 points) Find the values of $a$ and $b$ that make the function

$$f(x) = \begin{cases} 3 & \text{if } x = 1 \\ ax^2 - bx + 3 & \text{if } 1 < x < 2 \\ 2x - a + b & \text{if } 2 \leq x < 3 \\ 6 & \text{if } x = 3 \end{cases}$$

continuous on the closed interval $[1, 3]$. (Use limits to justify your steps)

10. (4-points) Given the function $f(x) = \dfrac{2x^2 + kx - 14}{x - 2}$, where $k$ is a constant, find $k$ such that $x = 2$ is a removable discontinuity of $f$. (Give reasons to your steps).