King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences

Math 202

Final Exam

Semester II, 1998–99 (982)

Name: ______________________________ ID #: ________________

Section (circle one): # 3 (8:00 – 8:50) # 4 (9:00 – 9:50) FORM(1)

Problem		points
1		10
2		13
3		12
4		12
5		18
6		4
7		4
8		11
9		4
10		4
11		4
12		4
Total:		100

Problem # 1 (Show all your work) (10 points)

Find two linearly independent solutions of the system $X' = \begin{bmatrix} 5 & 1 \\ -2 & 3 \end{bmatrix} X.$

$X_1(t) =$ ____

$X_2(t) =$ ____

The general solution is $X(t) = c_1 X_1(t) + c_2 X_2(t)$

Problem # 2. (Show all your work) (13 points)

Given that $\lambda_1 = \lambda_2 = \lambda_3 = 1$ are the eigenvalues of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & -1 \\ 0 & 1 & 0 \end{bmatrix}$. Find three linearly independent solutions of the system $X' = AX$.

$X_1(t) =$ ________________________________

$X_2(t) =$ ________________________________

$X_3(t) =$ ________________________________

The general solution is $X(t) = c_1X_1(t) + c_2X_2(t) + c_3X_3(t)$

Problem # 3. (Show all your work) (12 points)

Solve the given system

$$2x' - 5x + y' = e^t$$

$$x' - x + y' = 5e^t$$

$x(t) =$ ____________________

$y(t) =$ ____________________

Problem # 4 (Show all your work) (12 points)

Consider the following differential equation

$$xy'' + (x-6)y' - 3y = 0.$$

Obtain two linearly independent series solutions about the regular singular point $x_0 = 0$.

$y_1(x) =$ ______________________________ Valid ____________

$y_2(x) =$ ______________________________ Valid ____________

Problem # 5 (Show all your work) (18 points)

Consider the following differential equation

$$xy'' + 2y' - xy = 0.$$

Obtain two linearly independent series solutions about the regular singular point $x_0 = 0$.

$y_1(x) =$ ____ Valid ____________

$y_2(x) =$ ____ Valid ____________

Problem # 6 (Fill in the blanks) (4 points)

Determine all the singular points and the ordinary points of the following differential equation:

$$x^2(x+2)^2y'' + (x^2-4)y' + 2y = 0.$$

Classify each singular point as regular or irregular.

Ordinary points are: ____________________

Regular singular points are: ____________________

Irregular singular points are: ____________________

Problem # 7 (Fill in the blanks) (4 points)

Find the indicial equation and the indicial roots of:

$$xy'' + 2y' - xy = 0.$$

Given that $x_0 = 0$ is a regular singular point,

Indicial equation is ____________________

Indicial roots: $r_1 =$__________, $r_2 =$ __________

$r_1 - r_2 =$____________________

Problem # 8 (Fill in the blanks) (11 points)

Without solving, classify each of the following equation as to:

(a) Separable (b) homogeneous (c) exact (d) linear in x (e) linear in y (f) Bernoulli and (g) Ricatti

(I) $y' - 4 = 5y + y^2$ __________, __________, __________

(II) $y' = \dfrac{x-y}{x}$ __________, __________, __________

(III) $xyy' + y^2 = 2x$ __________, __________, __________

(IV) $\left(x^2 + \dfrac{2y}{x}\right)dx = (3 - \ln x^2)dx$ __________, __________, __________

(V) $2xyy' + y^2 = 2x^2$ __________, __________, __________

Problem # 9 (Circle the correct answer) (4 points)

If $y(x)$ is the solution of the following initial value problem:

$$xy' + y = \frac{1}{y^2}, \quad y(1) = 2,$$

then $y(2) =$

(a) $\frac{\sqrt[3]{15}}{2}$ (b) $\frac{\sqrt[3]{34}}{3}$ (c) $\frac{\sqrt[3]{1007}}{10}$ (d) $\frac{\sqrt[3]{71}}{4}$ (e) None of the above.

Problem # 10 (Circle the correct answer) (4 points)

If $y(x)$ is the solution of the following initial value problem:

$$y'' + 2y' + y = 4x^2 - 3, \quad y(0) = 21, \quad y'(0) = -16,$$

then $y(2) =$

(a) 9 (b) 41 (c) 5 (b) 69 (b) 41 (e) None of the above.

Problem # 11 (Circle the correct answer) (4 points)

The general solution of

$$y'' + 2y' + y = \frac{e^{-x}}{x} \quad \text{is}$$

(a) $y = c_1 \cos x + c_2 \sin x + x \ln x$

(b) $y = c_1 \cos x + c_2 \sin x + xe^{-x} \ln x$

(c) $y = c_1 \cos x + c_2 \sin x + xe^{x}$

(d) $y = c_1 \cos x + c_2 \sin x - xe^{-x} \ln x$

(d) None of the above.

Problem # 12 (Circle the correct answer) (4 points)

If $y(x)$ is the solution of the following initial value problem:

$$x^2y'' - 2xy' + 2y = 0, \quad y(1) = 0, \quad y'(1) = 1,$$

then $y(4) =$

(a) 2 (b) 6 (c) 12 (d) 20 (e) None of the above.