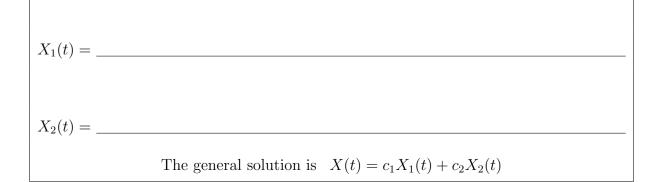
# King Fahd University of Petroleum and Minerals Department of Mathematical Sciences Math 202 Final Exam Semester II, 1998–99 (982)

Name:	ID #:	
Section (circle one): $\# 3 (8:00 - 8:50)$	$\# \ 4 \ (9{:}00 - 9{:}50)$	FORM(1)

Problem	points
1	10
2	13
3	12
4	12
5	18
6	4
7	4
8	11
9	4
10	4
11	4
12	4
Total:	100

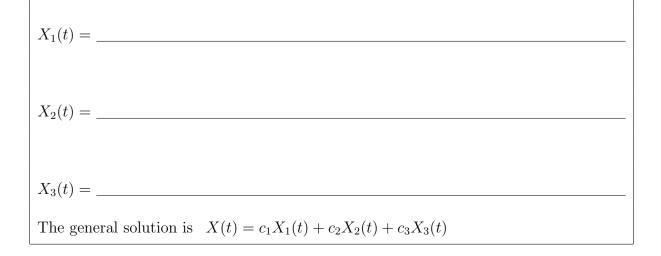
(10 points)

Find two linearly independent solutions of the system  $X' = \begin{bmatrix} 5 & 1 \\ & \\ -2 & 3 \end{bmatrix} X.$ 



Given that  $\lambda_1 = \lambda_2 = \lambda_3 = 1$  are the eigenvalues of the matrix  $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & -1 \\ 0 & 1 & 0 \end{bmatrix}$ . Find three linearly independent solutions of the system X' = AX.

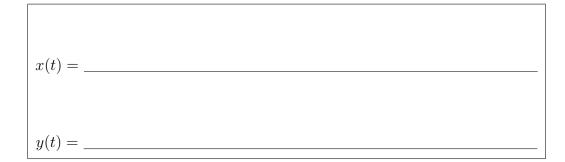
(13 points)



Solve the given system

$$2x' - 5x + y' = e^t$$
$$x' - x + y' = 5e^t$$

(12 points)



 $Problem \ \# \ 4 \ (\underline{Show \ all \ your \ work})$ 

Consider the following differential equation

$$xy'' + (x-6)y' - 3y = 0.$$

Obtain two linearly independent series solutions about the regular singular point  $x_0 = 0$ .

(12 points)

$y_1(x) = $	Valid
$y_2(x) = $	Valid

 $\mathbf{Problem} \ \# \ \mathbf{5} \ (\mathrm{Show \ all \ your \ work})$ 

Consider the following differential equation

$$xy'' + 2y' - xy = 0.$$

Obtain two linearly independent series solutions about the regular singular point  $x_0 = 0$ .

(18 points)

$y_1(x) = $	Valid
$y_2(x) = $	Valid

Determine all the singular points and the ordinary points of the following differential equation:

$$x^{2}(x+2)^{2}y'' + (x^{2}-4)y' + 2y = 0.$$

Classify each singular point as regular or irregular.

### Problem # 7 (Fill in the blanks)

Find the indicial equation and the indicial roots of:

$$xy'' + 2y' - xy = 0.$$

Given that  $x_0 = 0$  is a regular singular point,

Indicial equation is \_\_\_\_\_\_,  $r_2 = \_$ \_\_\_\_\_,  $r_1 - r_2 = \_$ \_\_\_\_\_\_

## Problem # 8 (Fill in the blanks)

(11 points)

(4 points)

Without solving, classify each of the following equation as to:

(a) Separable (b) homogeneous (c) exact (d) linear in x (e) linear in y (f) Bernoulli and (g) Ricatti

(4 points)

### Problem # 9 (Circle the correct answer)

If y(x) is the solution of the following initial value problem:

$$xy' + y = \frac{1}{y^2}, \quad y(1) = 2,$$

then y(2) =

(a) 
$$\frac{\sqrt[3]{15}}{2}$$
 (b)  $\frac{\sqrt[3]{34}}{3}$  (c)  $\frac{\sqrt[3]{1007}}{10}$  (d)  $\frac{\sqrt[3]{71}}{4}$  (e) None of the above.

#### Problem # 10 (Circle the correct answer)

If y(x) is the solution of the following initial value problem:

$$y'' + 2y' + y = 4x^2 - 3$$
,  $y(0) = 21$ ,  $y'(0) = -16$ ,  
then  $y(2) =$   
(a) 9 (b) 41 (c) 5 (b) 69 (b) 41 (e) None of the above.

#### Problem # 11 (Circle the correct answer)

The general solution of

$$y'' + 2y' + y = \frac{e^{-x}}{x}$$
 is

(a) 
$$y = c_1 \cos x + c_2 \sin x + x \ln x$$

(b)  $y = c_1 \cos x + c_2 \sin x + x e^{-x} \ln x$ 

(c) 
$$y = c_1 \cos x + c_2 \sin x + xe^x$$

- (d)  $y = c_1 \cos x + c_2 \sin x x e^{-x} \ln x$
- (d) None of the above.

#### Problem # 12 (Circle the correct answer)

If y(x) is the solution of the following initial value problem:

$$x^{2}y'' - 2xy' + 2y = 0, \quad y(1) = 0, \quad y'(1) = 1,$$

then y(4) =

(a) 2 (b) 6 (c) 12 (d) 20 (e) None of the above.

(4 points)

12

(4 points)