Name: KEY	ID: KEY	Sec: 2 , 3 serial:

MATH 202 **Term 052** **QUIZ 2 (form A)**

1) Determine whether the differential equation is separable or non-separable. (show your work)

(I)

$$(e^x + e^{-x})y' - y^2 = 0$$

$\Rightarrow (e^x + e^{-x})y' = y^2$

$\Rightarrow y' = \dfrac{y^2}{e^x + e^{-x}}$

$y' = \underbrace{(y^2)}_{g(y)}\underbrace{\left(\dfrac{1}{e^x + e^{-x}}\right)}_{f(x)} \Rightarrow$ separable

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(II)

$$y'(xy - 2x + 4y - 8) = (xy + 3x - y - 3)$$

$y'[x(y-2) + 4(y-2)] = [x(y+3) - (y+3)]$

$y'[y-2](x+4) = (y+3)[x-1]$

$y' = \dfrac{(y+3)(x-1)}{(y-2)(x+4)} \Rightarrow y' = \left(\dfrac{y+3}{y-2}\right)\left(\dfrac{x-1}{x+4}\right)$

$\Rightarrow$ separable

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(III)

$$e^{xy}\,y\,dy = (1 + e^{-2x})\,dx$$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{1 + e^{-2x}}{e^{xy}\,y}$

$\Rightarrow y' = \dfrac{1 + e^{-2x}}{y e^{xy}}$. note that LHS $\neq f(x)g(y)$

$\Rightarrow$ non-separable

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2) Solve the given linear differential equation (show all your work)
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KEY

$$(x+2)^2 y' = 5 - 8y - 4xy$$

$\Rightarrow (x+2)^2 y' = 5 - 4(2+x)y$

$(x+2)^2 y' = 5 - 4(x+2)y$

$\Rightarrow (x+2)^2 y' + 4(x+2)y = 5$

divid by $(x+2)^2$

$\Rightarrow \boxed{y' + \frac{4}{(x+2)} y = \frac{5}{(x+2)^2}}$ (1) is linear DE (5 pts)

with $p(x) = \frac{4}{x+2}$, $f(x) = \frac{5}{(x+2)^2}$

integrating factor $= y_1(x) = e^{\int p(x)dx} = e^{\int \frac{4}{x+2} dx} = e^{4\ln(x+2)}$

$= e^{\ln(x+2)^4} = (x+2)^4$

$\boxed{y_1(x) = (x+2)^4}$ (5 pts)

multiply equation (1) by $(x+2)^4$

$(x+2)^4 y' + 4(x+2)^3 y = 5(x+2)^2$

which becomes

$\frac{d}{dx}\left[(x+2)^4 y\right] = 5(x+2)^2$

integrate both sides

$(x+2)^4 y = \int 5(x+2)^2 dx$ (5 pts)

$\Rightarrow (x+2)^4 y = \frac{5}{3}(x+2)^3 + c \Rightarrow \boxed{y = \frac{5}{3}\frac{1}{(x+2)} + \frac{c}{(x+2)^4}}$ (5 pts)

Aside:

$\int (x+2)^2 dx =$

let $u = x+2$

$du = dx$

$= \int u^2 du = \frac{1}{3}u^3 + c$

$= \frac{1}{3}(x+2)^3 + c$