## Course Proposal

# Computability and Complexity 

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The Chairman<br>Department of Mathematics and Statistics<br>King Fahd University of Petroleum and Minerals

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Dear Chairman,
Please give your consideration to to enclosed course proposal. The title of the course is Computability and Complexity and is intended to run for the first time in the 082 semester. The course will cover the basics of computability theory from a mathematical perspective, and will include modules on Kolmogorov complexity and resource-bounded computation.

The students will be introduced to the the fundamental mathematical work of Alan Turing and Kurt Gödel amongst others, and will be shown some of the most interesting theorems of mathematical logic in the $20^{\text {th }}$ century. They will also learn about the famous unsolved problem $\mathrm{P}=\mathrm{NP}$ ? and obtain a appreciation for some of the issues involved in polynomial time computation. We will also touch on the concept of randomness and Komolgorov complexity - a research area in which I am both interested and active.

This course is intended to prepare students for further study in the area of mathematical computability theory, but for those students who do not continue in this direction of study, the course will provide an appreciation of some of the academic issues behind modern computing.

At first I am asking that this course be accepted for the degree of Bachelor of Science in Mathematics, but if we manage to attract enough computer science students, I will apply for it also to be counted as a core course for computer science degrees. I hope that this course will go some way to convincing students interested in computer science, that the study of pure mathematics is also an option for them.

Sincerely,

## Stephen Binns

Assistant Professor
Department of Mathematics and Statistics
KFUPM

## Proposed Course Specification: Computability and Complexity

| 0. Department | Department of Mathematics and Statistics |
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| A. Course Identification and General Information |  |
| 1. Course title | Computability and Complexity |
| 2. Credit hours | 3 |
| 3. Program | BS Mathematics |
| 4. Faculty | Dr Stephen Binns |
| 5. Level | 4th semester and above |
| 6. Prerequisites | MATH 232 or ICS 254 |
| 7. Corequisites | Nil |
| 8. Location | Main campus |
| B. Aims and Objectives |  | | 1. Summary of main learning | An introduction to the modern mathematical <br> theory behind computabilty and algorithmic complexity. <br> outcomes |
| :--- | :--- |
|  | The students will be exposed to the basic concepts and <br> techniques needed to continue with study of <br> theoretical computer science and mathematical <br> computability theory. For students who will not continue <br> further in this area, it is intended to give an appreciation <br> of the theory and issues underlying contemporary <br> computing and information technology. |
| 2. Course development plans | The course will be initially offered to those students <br> taking a BS in mathematics, however we intend to <br> attract computer science students as well. It is expected <br> that a variety of web-based material will be used <br> including Java applets simulating Turing machines. <br> Some use of actual computer coding may be appropriate <br> depending on the students' capabilities in these areas. <br> A WebCT page will be set up if the class size warrants it. <br> The chosen textbook lends itself to self-study and, <br> again depending on class size, some project work, <br> including class presentations, is possible. |

## C. Course Description

Proposed Bulletin description: The course will consist of the basics of mathematical computability theory including Turing machines, computable sets and languages, computable enumerability and decidability. Fundamental theorems such as the Recursion Theorem will be proved. The decidability of logical theories will be addressed and the proof of Gödel's famous Incompleteness Theorem will be sketched. The second half of the course will cover Algorithmic complexity theory and the major open problem $\mathrm{P}=\mathrm{NP}$ ? will be dealt with in detail.

| 1. Topics to be covered | See attached syllabus. |
| :--- | :--- |
| 2. Course Components | The course will consist of 3 lectures per week, <br> two major exams and a final exam. |
| 3. Additional private study <br> or learning hours | The students will be expected to spend an average <br> of 3 hours per week on homework. In addition there <br> will be two projects to be completed in the semester. <br> Approximately 5 hours to be spent on each project. |
| 4. Development of <br> Learning Outcomes in <br> Domains of Learning | a. Knowledge |
| (i) Knowledge to be acquired | Basics of computability Theory, Complexity Theory <br> and Randomness. Open problems especially P=NP? <br> will be explained. An understanding will be obtained <br> of the similarity between certain types of algorithms <br> with respect to computational feasibility. |
| (ii) Teaching strategies | Lectures will be the primary method of instruction. <br> Each new topic will be motivated at first asking an <br> open-ended question. The students will propose answers <br> which will be critiqued by the instructor and the class. <br> An introduction will then be given to the standard <br> mathematical analysis of the question. More details <br> will be added in subsequent lectures and questions arising <br> from the analysis will be raised and dealt with. Along the <br> way students will be expected to answer basic questions <br> based on the mathematical analysis |
| (iii) Methods of assessment | The assessment will be based on exams, projects and <br> quizzes. The projects will aim to evaluate students' <br> understanding as well as give them an opportunity to <br> investigate more thoroughly issues arising from the <br> material presented in lectures. <br> Quizzes will test that students' current knowledge of the <br> material, and exams will test the students' overall <br> understanding of the material. All evaluation methods <br> will test both understanding and problem-solving <br> ability. |


| C. Course Description cont. |  |
| :---: | :---: |
| b. Cognitive skills |  |
| (i) Cognitive skills to be developed | The ability to reason logically and precisely about a well-defined mathematical subject. <br> The ability to express arguments in the language of mathematics and computer science. <br> The ability to argue cogently with mathematical symbolism. <br> The development of the ability to think informally but accurately about the fundamental intuitive concepts of computability and complexity |
| (ii) Teaching strategies | The students will be expected to produce formal mathematical proofs. These will be corrected and returned to the student for improvement if necessary. It is important that the student learns to internalise the requirements of a correct mathematical argument, and feedback will be given through homework correction and review of quizzes and exams. Opportunity will be given for students to improve their homework grades by reviewing and correcting their own work. |
| (iii) Methods of assessment | Quizzes and exams will contain a mixture of questions designed to assess both general understanding and problem solving ability. True/false questions and short explanation questions will be used to evaluate a student's intuition and understanding. More detailed questions will be used to evaluate accuracy and logical argument. |
| c. Interpersonal skills and responsibility | Some group work may be assigned but in general the students will work individually. |
| (i) Skills to be developed |  |
| (ii) Teaching strategies |  |
| (iii) Methods of assessment |  |
| (d) Communication Information technology and numerical skills |  |
| (i) Skills to be developed | Professional communication of ideas and arguments. |
| (ii) Teaching strategies | A WEBCT page will be created with a chat-room to discuss homework and Projects |
| (iii) Methods of assessment | No direct assessment of these skills |
| (e) Psychomotor skills | Not Applicable |
| (i) Skills to be developed |  |
| (ii) Teaching strategies |  |
| (iii) Methods of assessment |  |
| 6. Schedule of assessment tasks |  |


| D. Student Support |  |
| :---: | :---: |
| 1. Availability of faculty for consultation and advice | The lecturer will be available for office hours on Sundays and Tuesdays, and on Saturdays, Mondays, and Wednesdays by appointment. After-hours help can be given via WebCT. |
| E. Learning Resources |  |
| 1. Required texts | Introduction to the Theory of Computation (2nd Ed.) by Michael Sipser, Course Technology (2005) |
| 2. Essential references |  |
| 3. Recommended books and reference material | A variety of extra books and material exist in the main library. Minimal use of these will be required as the proposed textbook is very comprehensive. |
| 4. Electronic materials |  |
| 5. Other materials |  |
| F. Facilities required |  |
| 1. Accommodation | One lecture room |
| 2. Computing resources | Projection facilities for electronic slides |
| 3. Other resources |  |
| G. Course evaluation and Improvement Processes |  |
| 1. Strategies for obtaining student feedback on quality of teaching. | Standard course evaluation form. Anonymous feedback form on WebCT. <br> Detailed course evaluation by students. |
| 2. Other strategies for evaluation of teaching | Review by the lecturer of the amount and nature of the material covered. |
| 3. Processes for improvement of teaching | Attending advanced WebCT seminar to improve knowledge and delivery of on-line teaching possibilities. |
| 4. Processes for verifying standards of student achievement | Comparison of grades with similar level mathematics courses. |
| 5. Action planning for improvement | Examination of on-line student feedback will lead to a review of the amount of material covered and a resulting adjustment in future offerings of this course. In particular, the section on Kolmogorov complexity can either be expanded or eliminated if required by time constraints. |

## Proposed Syllabus - Computability and Complexity

Section numbers refer to the proposed textbook:
Introduction to the theory of Computation 2nd edition by Michael Sipser.
PWS Publishing Company 2005.

| Week | Topic | Section | Topic | Hours |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Computability | $\begin{aligned} & 3.1 \\ & 3.2 \end{aligned}$ | Turing Machines Variants of Turing machines | 3 |
| 2 | Decidability | $\begin{aligned} & 3.3 \\ & 4.1 \\ & 4.2 \\ & \hline \end{aligned}$ | Algorithms and the Church-Turing Thesis Decidable Sets <br> Universal Turing Machines | 3 |
| 3 | Undecidable Problems | $\begin{aligned} & 4.2 \\ & 5.2 \\ & 5.2 \end{aligned}$ | The Halting Problem <br> Reducibility <br> Post Correspondence Problem | 3 |
| 4 | Turing Recognisable Languages | 5.3 | Mapping reducibility Computably enumerable sets Turing Completeness | 3 |
| 5 | Fundamental theorems of Computability Theory | 6.1 | The Enumeration Theorem The Parameter Theorem The Recursion Theorem | 3 |
| First exam |  |  |  |  |
| 6 | Logical theories and Decidability | $\begin{aligned} & \hline 6.2 \\ & 6.2 \\ & 6.2 \end{aligned}$ | Formulas and Proofs <br> Decidable Theories <br> Gödel's Incompleteness Theorem | 3 |
| 7 | Information and Kolmogorov Complexity | $\begin{aligned} & 6.3 \\ & 6.3 \\ & 6.3 \end{aligned}$ | Descriptions <br> Kolmogorov complexity <br> Compressibility and Randomness | 3 |
| 8 | Complexity Theory | $\begin{aligned} & 7.1 \\ & 7.1 \\ & 7.2 \\ & \hline \end{aligned}$ | Bounded Resources Big-O and Little-O notation Polynomial-time computation | 3 |
| 9 | Nondeterministic Computation | $\begin{aligned} & \hline 3.2 \\ & 7.3 \\ & 7.3 \end{aligned}$ | Nondeterministic Turing machines <br> The Class NP $\mathrm{P} \neq \mathrm{NP} ?$ | 3 |


|  |  | Second exam |  |  |
| :---: | :--- | :--- | :--- | :---: |
| 10 | NP completeness | 7.4 | Polynomial time reducibility |  |
|  |  | 7.4 | NP complete problems |  |
| 11 | Cook-Levin Theorem | 7.4 | SAT |  |
|  |  | 7.4 | SAT |  |
| 12 | Other NP-complete problems | 7.4 | SAT NP Complete | 3 |
|  |  | 7.5 | 3-SAT | CLIQUE |
| 13 | Space complexity | 7.5 | HAMPATH | 3 |
|  |  | 7.5 | VERTEX-COVER |  |
| 14 | PSPACE problems | 8.1 | Space complexity classes | 3 |
|  |  | 8.2 | Savitch's Theorem |  |

## List of American Schools Using the Proposed Textbook:

| San Jose State Univ | California State Poly Univ |
| :--- | :--- |
| California State Univ | Univ of California |
| California State Univ | Dixie Jr Coll |
| Univ of Nevada | Portland State Univ |
| U of Alaska - Anchorage | Univ of Oregon |
| Washington State Univ | University Of Washington |
| University Washington | Boise State Univ |
| Lewis-Clark State College | Montana State University |
| Montana Tech | Washington State University |
| Brigham Young Univ | BYU IDAHO |
| Idaho State University | Utah Valley State College |
| California State Univ | Univ of Nevada |
| California State Univ | Texas A\&M University |
| Community College Of Denver | University Of Wyoming |
| Univ of Texas at Dallas | Baylor Univ |
| University Of Texas | Univ of Texas - Pan American |
| El Paso CC | New Mexico State University |
| Univ of New Mexico | University Of Texas |
| University Of Houston | Arizona State Univ |
| Univ of Arizona | LA Tech College |
| Louisiana State Univ | McNeese State Univ |
| Southern Univ A\&M Coll | Univ of Louisiana |
| Univ of New Orleans | OK State Univ |
| Univ of Arkansas | Univ of Central Oklahoma |
| Arkansas State Univ | Columbia College |
| Univ of Missouri | Washington Univ in St Louis |
| Univ of Missouri | Avila College |
| Wichita State Univ | Creighton University |
| Univ of Nebraska | Univ of Minnesota |
| Metropolitan State Univ | Univ of Iowa |

