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On Principle of Equicontinuity

(Recommended by Prof. E. Dshalalow)

The main purpose of this paper is to prove some results of uniform boundedness principle type without the use of Baire's category theorem in certain topological vector spaces; this provides an alternate route and important technique to establish certain basic results of functional analysis. As applications, among other results, versions of the Banach—Steinhaus theorem and the Nikodym boundedness theorem are obtained.

Обоснованы некоторые результаты типа принципа однородной ограниченности без использования теоремы категории Байера в некоторых топологических векторных пространствах, что обеспечивает альтернативный способ и важную методику для получения результатов функционального анализа. Получены также версии теоремы Банаха — Штейнхауза и теоремы ограниченности Никодима.

K e y w o r d s: principle of equicontinuity, Banach—Steinhaus theorem, locally convex space, thick set.

1. Introduction. The classical uniform boundedness principle asserts: if a sequence $\{f_n\}$ of continuous linear transformations from a Banach space X into a normed space Y is pointwise bounded, then $\{f_n\}$ is uniformly bounded. The proof of this result is most often based on the Baire's category theorem (e. g. see Theorem 4.7—3 [18] and Theorem 3.17 [26]); the interested reader is referred to [10] for a new approach in this context. Several authors have sought proof of this type of results without Baire's theorem in various settings (see, for example, [4, 16], [23], [27]).

In 1933, Nikodym [21] proved: If a family M of bounded scalar measures on a σ-algebra **A** is setwise bounded, then the family M is uniformly bounded. This result is a striking improvement of the uniform boundedness principle in the space of countably additive measures on **A**; a Baire category proof of this theorem may be found in [9, IV. 9.8, p. 309]. Nikodym theorem has received a great deal of attention and has been generalized in several directions [5, 8, 19, 20, 28];

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