Random Coincidence Point Theorem in Fréchet Spaces with Applications

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We proved a random coincidence point theorem for a pair of commuting random operators in the setup of Fréchet spaces. As applications, we obtained random fixed point and best approximation results for *-nonexpansive multivalued maps. Our results are generalizations or stochastic versions of the corresponding results of Shahzad and Latif [2000], Khan and Hussain [2000], Tan and Yaun [1997] and Xu[1991].

Key Words: Random fixed point, Random coincidence point, *-nonexpansive map, Opial's condition, Fréchet space.

1. Introduction

Probabilistic functional analysis is an important mathematical discipline because of its applications to probabilistic models in applied problems. Random operator theory is needed for the study of various classes of random equations. The study of random fixed point theorems was initiated by the Prague school of probabilists in the 1950s. The interest in this subject enhanced after publication of the survey paper by Bharucha-Reid [4]. Random fixed point theory has received much attention in recent years (see for example, Beg and Shahzad [2], Khan and Hussain [8, 9,10], Lin [12], Sehgal and Singh [16], Shahzad and Khan [17], Tan and Yaun [19] and Xu [21]). Recently, Shahzad and Latif [18] established a random coincidence point theorem for a pair of commuting random operators defined on a separable weakly compact star-shaped subset M of a Banach space. In this paper, we extend Shahzad and Latif's result to Fréchet spaces. We apply new result to prove random fixed point and best approximation theorems for a noncontinuous multivalued map namely

*-nonexpansive map defined on the subset M of a Fréchet space. Our work extends results of Beg and Shahzad [1], Khan and Hussain [8, 10] and Tan and Yaun [19] and provides stochastic version of the corresponding results of Carbone [5], Husain and Latif [6], Khan and Hussain [7], Sahney et. al [14] and Xu [20].

2. Preliminaries

Let X be a complete metric space and $(\Omega, \sum)$ a measurable space. Let $CB(X)$ and $K(X)$ denote the families of all nonempty bounded closed subsets and all nonempty compact subsets of X respectively. A mapping $T : \Omega \to CB(X)$ is called measurable if for any open subset C of X,

$$T^{-1}(C) = \{\omega \in \Omega : T(\omega) \cap C \neq \phi\} \in \sum.$$

A mapping $\xi : \Omega \to X$ is said to be a measurable selector of a measurable mapping $T : \Omega \to CB(X)$ if ξ is measurable and for any $\omega \in \Omega, \xi(\omega) \in T(\omega)$. A mapping $T : \Omega \times X \to CB(X)($ resp. $f : \Omega \times X \to X)$ is called a random operator if for any $x \in X, T(., x)($ resp. $f(., x))$ is measurable. A measurable mapping $\xi : \Omega \to X$ is called a random fixed point of a random operator $T : \Omega \times X \to CB(X)($ resp. $f : \Omega \times X \to X)$ if for every $\omega \in \Omega, \xi(\omega) \in T(\omega, \xi(\omega))($ resp. $f(\omega, \xi(\omega)) = \xi(\omega))$. A measurable mapping $\xi : \Omega \to X$ is a random coincidence point of random operators $T : \Omega \times X \to CB(X)$ and $f : \Omega \times X \to X$ if for every $\omega \in \Omega, f(\omega, \xi(\omega)) \in T(\omega, \xi(\omega))$. A Fréchet space X satisfies Opial's condition if for every sequence $\{x_n\}$ in X weakly convergent to $x \in X$, the inequality

$$\liminf_{n\to\infty} d(x_n, x) < \liminf_{n\to\infty} d(x_n, y)$$

holds for all $y \neq x$. Every Hilbert space and the space $l_p (1 \leq p < \infty)$ satisfy Opial's condition. A subset M of X is said to be star-shaped with respect to $q \in M$ if $\{(1 - t)x + tq : 0 \leq t \leq 1\} \subset M$ for each $x \in M$. The star-shaped subsets include the convex subsets as a proper subclass. A multivalued map $T : X \to CB(X)$ is said to be demiclosed if for every sequence (net) $\{x_n\}$ in X and $y_n \in T(x_n), n = 1, 2, \ldots$, such that $\{x_n\}$ converges weakly to x and $\{y_n\}$ converges strongly to y, we have $x \in X$ and $y \in T(x)$. A mapping $f : X \to X$ is called weakly continuous if $\{x_n\}$ converges weakly to x implies $\{f(x_n)\}$ converges weakly to $f(x)$. Also, a mapping f on a convex set M is called affine if $f(tx + (1 - t)y) = tf(x) + (1 - t)f(y)$ for all $x, y \in M$ and $0 \leq t \leq 1$.

A mapping $T : M \to 2^X$ is said to be (i) nonexpansive if for all $x, y \in M$,

$$H(T(x), T(y)) \leq d(x, y)$$

where H is the Hausdorff metric on $CB(X)$ induced by the metric d.

(ii) *-nonexpansive (cf. [6, 20]) if for all $x, y \in M$ and $u_x \in Tx$ with $d(x, u_x) = d(x, Tx) = \inf\{d(x, z) : z \in Tx\}$, there exists $u_y \in T_y$ with $d(y, u_y) = d(y, T_y)$ such that

$$d(u_x, u_y) \leq d(x, y)$$

(iii) upper semicontinuous (lower semicontinuous) if for any closed (open) subset B of X, $T^{-1}(B) = \{x \in M : T(x) \cap B \neq \phi\}$ is closed (open).

(iv) continuous if it is both upper semicontinuous and lower semicontinuous.

For each $x \in M$, we follow Xu [20] to define the set (possibly empty)

$$P_T(x) = \{u_x \in Tx : d(x, u_x) = d(x, Tx)\}$$

Recall that the metric projection $P : X \to 2^M$ is defined by
$P_M(x) = \{m \in M : d(x, m) = d(x, M)\}$.

The concept of a *-nonexpansive multivalued mapping is different from continuity of the map as is clear from the following:

Example 1.1. Let $T : [0, 1] \to 2^{[0,1]}$ be a multivalued map defined by

$$Tx = \begin{cases} \{\frac{1}{2}\}, & x \in [0, \frac{1}{2}) \cup (\frac{1}{2}, 1] \\ \left[\frac{1}{4}, \frac{3}{4}\right], & x = \frac{1}{2} \end{cases}$$

Then $P_T(x) = \left\{\frac{1}{2}\right\}$ for every $x \in [0, 1]$. This implies that T is a *-nonexpansive map.

$$H(T(\frac{1}{3}), T(\frac{1}{2})) = H(\{\frac{1}{2}\}, [\frac{1}{4}, \frac{3}{4}]) = max\{0, \frac{1}{4}\} = \frac{1}{4} > \frac{1}{6} = |\frac{1}{3} - \frac{1}{2}|.$$

If $V_{\frac{1}{4}}$ is any small open neighbourhood of $\frac{1}{4}$, then the set

$$T^{-1}(V_{\frac{1}{4}}) = \{x \in [0, 1] : Tx \cap V_{\frac{1}{4}} \neq \phi\} = \{\frac{1}{2}\}$$

is not open. Thus T is neither nonexpansive nor lower semicontinuous. Note that $\frac{1}{2}$ is a fixed point of T.

Example 1.2. Let $X = \mathcal{R}^2$ be equipped with Euclidean norm and $M = \left\{(a, 0) : \frac{1}{\sqrt{2}} \leq a \leq 1\right\} \cup \{(0, 0)\}$. Define $T : M \to 2^X$ by

$$T(a, 0) = \begin{cases} (0, 1) & \text{if} \quad a \neq 0 \\ L = \text{ the line segment } [(0, 1), (1, 0)] & \text{if} \quad a = 0 \end{cases}$$

Then $P_T(a,0) = \{(0,1)\}$ for all $(a,0) \in M$ with $a \neq 0$ and $P_T(0,0) = \left(\frac{1}{2}, \frac{1}{2}\right)$. Clearly T is *-nonexpansive but not a nonexpansive multifunction (cf. [16], p. 537).

A random operator $T : \Omega \times M \to 2^X (resp. f : \Omega \times X \to X)$ is said to be continuous (weakly continuous, *-nonexpansive etc.) if for each $\omega \in \Omega, T(\omega, .)$ (resp. $f(\omega, .)$) is continuous (weakly continuous, *-nonexpansive etc.).

It is well known that a Hausdorff locally convex topological vector space X is metrizable if and only if X has a countable base of absolutely convex neighbourhoods of zero or, equivalently, X has a countable family of seminorms $\{p_n\}$ that generates the locally convex topology on X. We can always assume that $p_n \leq p_{n+1}, n \geq 1$. A function $d : X \times X \to \mathcal{R}^+ \cup \{0\}$ given by

$$d(x,y) = \sum_{n=1}^{\infty} \frac{c_n p_n(x-y)}{1 + p_n(x-y)}$$

for $x, y \in X$ with $c_n > 0$ and $\sum_{n=1}^{\infty} c_n < \infty$, defines a metric on X.

A subspace M of X is said to be quasi-Chebyshev if $P_M(x)$ is a nonempty and compact set in X for each x in X (cf. [10]).

Let $f : M \to X$ be a map. Then $T : M \to K(X)$ is called an f-nonexansive mapping if $H(T(x), T(y)) \leq d(f(x), f(y))$.

3. Results

Sahney et. al. [14] proved approximation results of very general nature in locally convex spaces. Recently random fixed point results in the context of a Fréchet space X have been studied by Shahzad and Khan in [17]. We establish random coincidence point, random fixed point and random best approximation results for multivalued maps defined on suitable subsets of a Fréchet space. Our results are improvements, generalizations or stochastic versions of several known results.

We shall need the following variant of Nadler's result.

Lemma 3.1. ([15], Lemma) Let X be a Fréchet space. If $A, B \in K(X)$, *then for each* $a \in A$, *there is a* $b \in B$ *such that* $d(a,b) \leq H(A,B)$.

Applying the above lemma, analogue of Lemma 3.1 due to Latif and Tweddle [11] is established in the following.

Lemma 3.2. *Let M be a nonempty weakly compact subset of a Fréchet X satisfying Opial's condition. Let $f : M \to X$ be a weakly continuous map and $T : M \to K(X)$ an f-nonexpansive multivalued map. Then $f - T$ is demiclosed.*

Proof. Let $\{x_a\}$ be a net in M and $y_\alpha \in (f - T)(x_\alpha)$ be such that $x_\alpha \longrightarrow x$ weakly and $y_\alpha \longrightarrow y$. Obviously $x \in M$ and $f(x_\alpha) \longrightarrow f(x) weakly$. Since $y_\alpha \in f(x_\alpha) - T(x_\alpha)$; therefore we have $y_\alpha = f(x_\alpha) - u_\alpha$, for some $u_\alpha \in T(x_\alpha)$. As $T(x)$ is compact, by Lemma 3.1, there is a $u_\alpha \in T(x)$ such that

$$d(u_\alpha, v_\alpha) \leq H(T(x_\alpha), T(x)).$$

The f-nonexpansiveness of T gives that

$$H(T(x_\alpha), T(x)) \leq d(f(x_\alpha), f(x)).$$

Thus

$$d(u_\alpha, v_\alpha) \leq d(f(x_\alpha), f(x)).$$

Passing to the limit with respect to α, we obtain

$$\begin{aligned} \liminf d(f(x_\alpha), f(x)) &\geq \liminf d(u_\alpha, v_\alpha) \\ &= \liminf d(f(x_\alpha), y_\alpha + v_\alpha). \end{aligned} \tag{1}$$

By compactness of $T(x)$, for a convenient subnet still denoted by $\{v_\alpha\}$, we have $v_\alpha \longrightarrow v \in T(x)$. Consequently (1) yields

$$\liminf d(f(x_\alpha), f(x)) \geq \liminf d(f(x_\alpha), y + v).$$

Since X satisfies Opial's condition and $f(x_\alpha) \to f(x)$ weakly, $f(x) = y + v$. Thus $y = f(x) - v \in f(x) - T(x)$, which proves that $f - T$ is demiclosed.

Theorem 3.3. ([2], Theorem 5.1). *Let (X, d) be a separable complete metric space, $T : \Omega \times X \to CB(X)$ a multivalued random operator, and $f : \Omega \times X \to X$ a continuous random operator such that $T(\omega, X) \subset f(\omega, X)$ for each $\omega \in \Omega$. If f and T commute and for all $x, y \in X$ and all $\omega \in \Omega$, we have*

$$H(T(\omega, x), T(\omega, y)) \leq kd(f(w, x), f(\omega, y)),$$

$k \in (0, 1)$, then T and f have a random coincidence point.

We shall follow the argument used by Shahzad and Latif [18] to prove the following random coincidence point theorem in the context of Fréchet spaces.

Theorem 3.4. *Let M be a separable weakly compact subset of a Fréchet space X which is star-shaped with respect to $q \in M$, and let $f : \Omega \times M \to M$ be a continuous affine random operator such that $f(\omega, M) = M$ and $f(\omega, q) = q$ for each $\omega \in \Omega$. Let $T : \Omega \times M \to K(M)$ be a multivalued random operator which commutes with f and for all $x, y \in M$ and all $\omega \in \Omega$, we have*

$$H(T(\omega, x), T(\omega, y)) \leq d(f(\omega, x), f(\omega, y)).$$

If one of the following conditions hold: either (a) *$(f - T)(\omega, .)$ is demiclosed at zero for each $\omega \in \Omega$ or (b)f is weakly continuous and X satisfies Opial's condition, then T and f have a random coincidence point.*

Proof. Choose a sequence $\{k_n\}$ of real numbers with $0 < k_n < 1$ and $k_n \to 0$ as $n \to \infty$. For each n, consider the random operator $T_n : \Omega \times M \to K(M)$ defined by

$$T_n(\omega, x) = k_n q + (1 - k_n)\, T(\omega, x).$$

Then,

$$\begin{aligned} H\left(T_n(\omega, x), T_n(\omega, y)\right) &= (1 - k_n)\, H(T(\omega, x), T(\omega, y)) \\ &\leq (1 - k_n)\, d(f(\omega, x), f(\omega, y)) \end{aligned}$$

for each $x, y \in M$ and each $\omega \in \Omega$. Since $T(\omega, M) \subset M = f(\omega, M)$, we have

$$T_n(\omega, M) \subset f(\omega, M)$$

for each $\omega \in \Omega$. Further, each T_n commutes with f, since for any $x \in M$ and $\omega \in \Omega$, we have

$$\begin{aligned} T_n(\omega, f(\omega, x)) &= k_n q + (1 - k_n)\, T(\omega, f(\omega, x)) \\ &= k_n f(\omega, q) + (1 - k_n)\, f(\omega, T(\omega, x)) \\ &= f\left(\omega, \{k_n q + (1 - k_n)\, T(\omega, x)\}\right) \\ &= f\left(\omega, T_n(\omega, x)\right). \end{aligned}$$

Since M is separable and weakly compact, the weak topology on M is a metric topology (cf. Rudin [13], p. 86). It follows that M is a complete metric space. Thus, by Theorem 3.3, there is a measurable map $\xi_n : \Omega \to M$ such that

$$f\left(\omega, \xi_n(\omega)\right) \in T_n\left(\omega, \xi_n(\omega)\right)$$

for each $\omega \in \Omega$. For each n, define $F_n : \Omega \to WK(M)$ by

$$F_n(\omega) = w - cl\left\{\xi_i(\omega) : i \geq n\right\},$$

where $WK(M)$ is the family of all nonempty weakly compact subsets of M and $w-cl$ denotes the weak closure. Define $F : \Omega \to WK(M)$ by $F(\omega) = \bigcap\limits_{n=1}^{\infty} F_n(\omega)$. As before, the weak topology on M is a metric topology. Then as in [9, proof of Theorem 2.3 or 17, Theorem 3.3] F is w-measurable and has a measurable selector ξ. This ξ is the desired random coincidence point of f and T. Indeed, fix $\omega \in \Omega$ arbitrarily. Then some subsequence $\{\xi_m(\omega)\}$ of $\{\xi_n(\omega)\}$ converges weakly to $\xi(\omega)$. Also there is some $u_m \in T(\omega, \xi_m(\omega))$ such that

$$f(\omega, \xi_m(\omega)) - u_m = k_m\left\{q - u_m\right\}.$$

The set M is bounded and $k_m \to 0$, it follows that $f(\omega, \xi_m(\omega)) - u_m \to 0$. Now $y_m = f(\omega, \xi_m(\omega)) - u_m \in (f - T)(\omega, \xi_m(\omega))$ and $y_m \to 0$. If (a) holds, then it follows that $f(\omega, \xi(\omega)) \in T(\omega, \xi(\omega))$. If (b) holds, then, by Lemma 3.2, $(f - T)(\omega, .)$ is demiclosed, and therefore, f and T have a random coincidence point ξ.

Theorem 3.4 yields a common fixed point theorem as follows.

Theorem 3.5. *Suppose that M, f, T, and q satisfy all the hypotheses of Theorem 3.4. If for each $\omega \in \Omega$*

$$f(\omega, x) \in T(\omega, x) \text{ implies the existence of } \lim_n f^n(\omega, x),$$

then T and f have a common random fixed point.

Proof: Verbatim repetition of the proof of Theorem 3.3 [18] and is omitted.

The following result concerning measurability of the map P_T will be needed.

Proposition 3.6. Let C be a separable closed subset of a complete metric space and $T : \Omega \times C \to 2^C$ is a compact valued measurable function. Then P_T is also a measurable function.

Proof: By Proposition 2.2 [3], the map $T : \Omega \times C \to 2^C$ is measurable if and only if for each x in C, the function $d(x, T(\omega, x))$ is measurable. Note that $P_T : \Omega \times C \to 2^C$ is a well-defined compact valued map such that $d(x, P_T(\omega, x)) \leq$

$d(x, u_x) = d(x, T(\omega, x)) \leq d(x, P_T(\omega, x))$ for all x in C and $\omega \in \Omega$. Thus for each x in $C, d(x, P_T(\omega, x))$ is measurable and hence P_T is measurable.

As an other applicaton of Theorem 3.4, we obtain the follwing random fixed point result for *-nonexpansive maps.

Theorem 3.7. Let M be a separable weakly compact star-shaped subset of a Fréchet space X and $T : \Omega \times M \to K(M)$ a *-nonexpansive random operator. Then T has a random fixed point if one of the following conditions holds:

(a) $I - T(\omega, .)$ is demiclosed at 0 for each $\omega \in \Omega$(I denotes the identity map).

(b) X satisfies Opial's condition.

Proof: Each set $T(\omega, x)$ being compact is proximinal and so $P_T : \Omega \times M \to 2^M$ is well defined. It follows from the definition of T is *-nonexpansive that P_T is nonexpansive (see proof of Theorem 2 [20]). The map P_T is compact valued as T is so. Therefore, for each $\omega \in \Omega$ and $x \in M$, we have by definition of P_T,

$$\begin{aligned} d\left(x, P_T(\omega, x)\right) &\leq d\left(x, u_x\right) = d(x, T(\omega, x)) \\ &\leq d\left(x, P_T(\omega, x)\right) \end{aligned} \tag{2}$$

Moreover P_T is a random operator by Proposition 3.6.

(a) Suppose $x_n \longrightarrow x_0$ weakly and $y_n \in I - P_T(\omega, x_n)$ such that $y_n \longrightarrow 0$ strongly. Note that $y_n \in I - P_T(\omega, x_n) \subseteq I - T(\omega, x_n)$ and $I - T(\omega, .)$ is demiclosed at 0 so $0 \in I - T(\omega, x_0)$. This implies that $x_0 \in T(\omega, x_0)$ and hence $d\left(x_0, T(\omega, x_0)\right) = 0$. By (2), $d\left(x_0, P_T(\omega, x_0)\right) = d(x_0, T(\omega, x_0))$. Thus $x_0 \in P_T(\omega, x_0)$ implies that $I - P_T(\omega, .)$ is demiclosed at 0 for each $\omega \in \Omega$. By Theorem 3.4 (a), P_T has a random fixed point which is also a random fixed point of T.

(b) In this case $I - P_T(\omega, .)$ is demiclosed at 0 for each $\omega \in \Omega$ and the result follows from (a).

The above theorem leads to the existence of random best approximation in the pair of corollaries to follow.

Corollary 3.8. Let M be a separable weakly compact star-shaped subset of a Fréchet space X and $F : \Omega \times M \to K(X)$ an upper semicontinuous random map. Suppose that $T : \Omega \times M \to 2^M$ defined by $T(\omega, x) = \bigcup\{P(y) : y \in F(\omega, x), d(F\omega, x), M) = d(y, M)\}$ is a compact valued *-nonexpansive random operator. Then there exists a measurable map $\xi : \Omega \to M$ such that $d(\xi(\omega), F(\omega, \xi(\omega)) = d(F(\omega, \xi(\omega)), M)$ for each $\omega \in \Omega$ provided either (a) or (b) of Theorem 3.7 holds.

Proof. Under each of the conditions (a) and (b), by Theorem 3.7, there exists a measurable map $\xi : \Omega \to M$ such that $\xi(\omega) \in T(\omega, \xi(\omega))$ for each $\omega \in \Omega$. Fix $\omega \in \Omega$ arbitrarily. Then for some y in $F(\omega, \xi(\omega))$ with $d(F(\omega, \xi(\omega)), M) = d(y, M), \xi(\omega) \in P(y)$. Now $d(\xi(\omega), F(\omega, \xi(\omega)) \leq d(\xi(\omega), y) = d(y, M) = d(F(\omega, \xi(\omega)), M) \leq d(\xi(\omega), F(\omega, \xi(\omega))$ implies that $d(\xi(\omega), F(\omega, \xi(\omega))) = d(F(\omega, \xi(\omega)), M)$ for each $\omega \in \Omega$.

Corollary 3.9. Let M be a separable weakly compact star-shaped subset of a Fréchet space X. Suppose that $F : \Omega \times M \to X$ is a continuous random operator and $PoF : \Omega \times M \to M$ is a nonexpansive random operator. Then the conclusion of Corollary 3.8 holds provided either (a) I-$PoF(\omega, .)$ is demiclosed at 0 for each $\omega \in \Omega$ or (b) X satisfies Opial's condition.

An other consequence of Theorem 3.4 provides the following result on invariant random best approximation (cf. Theorems 4 and 5 [1]).

Theorem 3.10. Let X be a Fréchet space and $T : \Omega \times X \to K(X)$ be a nonexpansive random operator such that for each $\omega \in \Omega, T(\omega, .)(u) = \{u\}$ for some $u \in X$. Let M be a nonempty $T(\omega, .)-$ invariant subset of X for each $\omega \in \Omega$. Assme that $D = P_M(u)$ is nonempty separable weakly compact and starshaped. Then u has a random best approximation $\xi : \Omega \to M$ which is also a random fixed point of T provided either (a) $I - T(\omega, .)$ is demiclosed at zero for each $\omega \in \Omega$ or (b) X satisfies Opial's condition.

Proof. Let $y \in D$. Then, $y \in M$ and $d(u, y) = d(u, M)$. Fix ω arbitrarily in Ω and let $x \in T(\omega, y) \subseteq M$. Then

$$\begin{aligned} d(x, u) &\leq H(T(\omega, y), T(\omega, u)) \\ &\leq d(y, u) = d(u, M). \end{aligned}$$

So we have $x \in D$. Thus $T(\omega, y) \subseteq D$. Hence $T(\omega, .)$ maps D into $K(D)$ for each $\omega \in \Omega$. Therefore, by Theorem 3.4, T has a random fixed point in $P_M(u)$.

Corollary 3.11. Let X be a Banach space, $CK(X)$ denote the family of all nonempty convex compact subsets of X and $T : \Omega \times X \to CK(X)$ be a nonexpansive random operator such that for each $\omega \in \Omega, T(\omega, .)(u) = \{u\}$ for some $u \in X$. Assume that for each $\omega \in \Omega, T(\omega, .)$ leaves a quasi-Chebyshev subspace M invariant. Then u has a random best approximation $\xi : \Omega \to M$ which is also a random fixed point of T.

Proof. The set $P_M(u)$ is nonempty and compact and hence separable. Moreover, $P_M(u)$ is convex. As in the proof of Theorem 3.10, $P_M(u)$ is $T(\omega, .)$-invariant for each $\omega \in \Omega$. Thus T has a random fixed point in $P_M(u)$ by Corollary 3.3 [3].

Remarks 3.12.

(i) Theorem 3.4 extends [3, Corollaries 3.1 and 3.2, 18, Theorem 3.2, 21, Theorem 1 (ii)].

(ii) Theorem 3.5 extends [8, Corollary 3.9, 9, Corollaries 2.4 and 2.12, 18, Theorem 3.3].

(iii) Theorem 3.7(a) generalizes Theorem 3.4 [19] to multivalued *-nonexpansive maps.

(iv) Theorem 3.7 (b) sets stochastic analogues of [6, Theorem 3.2, 20, Theorem 2] in Fr$\acute{e}$chet spaces. Further, it extends Corollary 3.5 [19] to *-nonexpansive maps.

(v) Theorem 3.7 (b) provides conclusions of Corollary 3.9 [8] and Theorem 2.11 [9] without the measurability of P_T in the framework of a Fr$\acute{e}$chet space and an arbitrary measure space.

(vi) Corollary 3.8 establishes a generalized stochastic version of Theorem 1 [7].

(vii) Corollary 3.9 generalizes Theorem 3 [12] and provides a stochastic version of Theorem 3 [5].

(viii) Theorem 3.10 gives multivalued random analogue of Corollaries 3.1, 3.4 and 3.6 (i) [14] (see also Corollary 3.9 [10]) while Corollary 3.11 extends Theorem 4[1], Corollary 3.3 [10] and corollary 3.3 [14].

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