Indian J. pure appl. Math., **31**(8) : 983-987, August 2000

BEST APPROXIMATION AND FIXED POINT RESULTS

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(*Received* 1 *September* 1999; *acceepted* 11 *November* 1999)

This note presents theorems on the existence of best approximations by making use of *-nonexpansive and continuous maps on certain classes of topological vector spaces. Several fixed point theorems are obtained as well.

Key Words : Best Approximation; Fixed Point; Spaces-Topological Vector; Maps-Nonexpensive; Maps-Continuous

We shall need following definitions :

Let M be a nonempty subset of a normed space E. An element $y \in M$ is called a best approximation to $x \in E$ if

$$\| x - y \| = d(x, M) = \inf \{\| x - m \| : m \in M\}.$$

The set of best M-approximations to x is denoted by $P(x)$ and is defined as

$$P(x) = \{m \in M : \| x - m \| = d(x, M)\}.$$

The mapping $P : E \to M$ is called metric projection.

In case $P(x)$ contains only one element for every $x \in E, M$ is called a Chebyshev set. A closed convex subset M of a Hilbert space is Chebyshev and projection map P is nonexpansive.

A Banach space E is said to satisfy Opial's condition if for each $x \in E$ and each sequence $\{x_n\}$ weakly converging to x, $\liminf \| x_n - x \| < \liminf \| x_n - y \|$ holds for all $y \neq x$ in E.

A multivalued map $f : M \to 2^M$ is called (*i*) weakly nonexpansive (cf. [5]) if given $x \in M$ and $u_x \in f(x)$ there is a $u_y \in f(y)$ for each $y \in M$ such that

$$\| u_x - u_y \| \leq \| x - y \|.$$

(*ii*) *-nonexpansive (cf. [5]) if for all x, y in M and $u_x \in f(x)$ with

$$\| x - u_x \| = d(x, f(x))$$

there exists $u_y \in f(y)$ with $\| y - u_y \| = d(y, f(y))$ such that $\| u_x - u_y \| \leq \| x - y \|$.