Part I: (9 points) MULTIPLE CHOICE QUESTIONS: (MCQ)
[Bubble the correct answer on the OMR sheet]

1. The equation $10^x = x$ has:

   a) no real solutions

   b) one real solution

   c) two real solutions

   d) three real solutions

2. If the turntable of a record player turns at 30 revolutions per minute, then the angular speed in radians per second is:

   a) π rad/sec

   b) 2π rad/sec

   c) 1 rad/sec

   d) 2 rad/sec

3. The expression $\left(\frac{1}{1-\cos\theta}+\frac{1}{1+\cos\theta}\right)\div\left(\frac{1}{1-\sin\theta}+\frac{1}{1+\sin\theta}\right)$ simplifies to:

   a) $\cot^2\theta$

   b) $\tan^2\theta$

   c) $\sec^2\theta$

   d) $\csc^2\theta$

4. The asymptote of the graph of the function $f(x)=e^{-x}+1$ is:

a) $y=1$

b) $y=0$

c) $y=e$

d) $x=1$

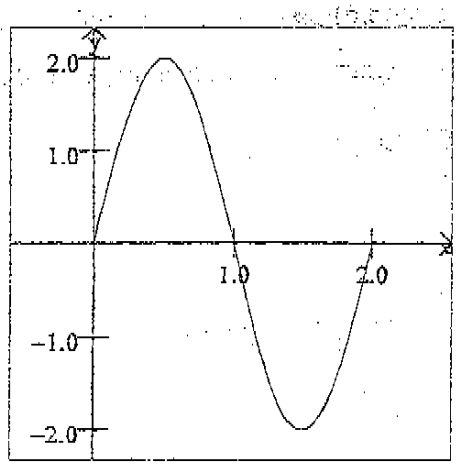
5. The adjacent figure represents for one period the graph of the function

a) $f(x)=2\sin \pi x$

b) $f(x)=-2\sin \pi x$

c) $f(x)=\sin \pi x$

d) $f(x)=2\sin x$



6. Let W be the wrapping function. For any real number t, the distance between $W(t)$ and $W(t+\pi)$ is:

a) 2

b) 1

c) $\frac{\pi}{2}$

d) 2π

Part II: (7 points) **[Fill in the blanks in the following questions]:**
[Show your steps]

1. The range of the function $f(x) = -2^x - 3$ is $(-\infty, -3)$

$2^x > 0 \Rightarrow -2^x < 0 \Rightarrow -2^x - 3 < -3$, i.e, $f(x) < -3$

2. The inverse function of the function $f(x) = \log_2(x-1)$ is $f^{-1}(x) = 2^x + 1$

$y = \log_2(x-1)$

$x = \log_2(y-1) \Rightarrow 2^x = y - 1 \Rightarrow y = 2^x + 1$

3. The measure of the complement of the angle $\theta = 40°29'$ is.....

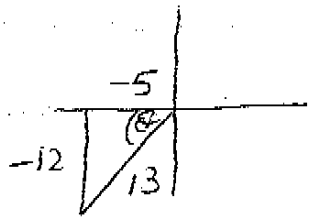
$\theta = 90° - 40° 29' = 49° 31'$

4. The length of the arc of a circle with radius 9 cm and central angle $\theta = 40°$ is 2π cm.

$S = r\theta = 9 * 40 * \frac{\pi}{180} = 2\pi$

5. If $\tan\theta = \frac{12}{5}$ and $\sec\theta < 0$, then $\sin(-\theta) = \frac{12}{13}$

$\sin(-\theta) = -\sin\theta = -\left(\frac{-12}{13}\right) = \frac{12}{13}$



6. The reference angle of the angle $-120°$ is $60°$

coterminal: $-120° + 360° = 240°$

reference: $240° - 180° = 60°$

7. The value of $2^{\frac{\ln 5}{\ln 2}}$ is ....5...

$2^{\log_2 5} = 5$

Part III: WRITTEN QUESTIONS
[Provide neat and complete solution to each question. Show necessary steps for full credit.]

1. (3 points) If $f(x) = e^{1+x} - e^{-x}$, then find the value of $f(3\ln 2)$.

$f(3\ln 2) = e^{1+3\ln 2} - e^{-3\ln 2} = e \cdot e^{3\ln 2} - e^{-3\ln 2}$

$= e \cdot e^{\ln 8} - e^{\ln \frac{1}{8}}$ —— (2 pts)

$= 8e - \frac{1}{8}$ —— (1 pt)

2. Find the solution set of each of the following equations:

a) (4 points) $\left(\sqrt{2}\right)^{(12x-8)} = 4\left(\frac{1}{2}\right)^{(5x-5)}$

$2^{6x-4} = 2^2 * 2^{-5x+5}$

$2^{6x-4} = 2^{-5x+7}$ —— 2pts

$6x-4 = -5x+7$ —— —— 1pt.

$11x = 11$ S.S = {1}

$x = 1$ —— —— 1pt.

b) (4 points) $\log(4-x) = \log(x+8) + \log(2x+13)$

$\log(4-x) = \log(2x^2+19x+104)$ —— 1pt.

$4-x = 2x^2+19x+104$ —— 1pt.

$2x^2+30x+100 = 0$

$x^2+15x+50 = 0$ —— 1pt.

$(x+10)(x+5) = 0$

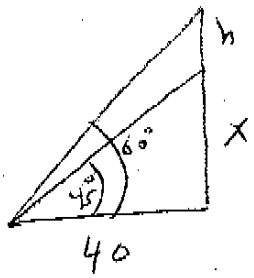
$x = -10$ or $x = -5$ ✓

x rejected

S.S = {-5} —— 1pt.

3. (3 points) The angle of elevation of the top of an unfinished tower at a point distant 40 meters from its base is $45°$. How much higher must be the tower raised so that the angle of elevation at the same point is $60°$?

$\tan 45° = \frac{x}{40} \Rightarrow x = 40$ —— 1pt.



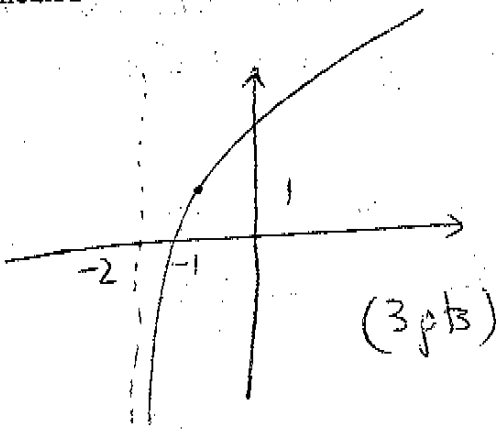
$\tan 60° = \frac{h+40}{40}$ —— 1pt.

$40\sqrt{3} = h + 40$

$h = 40\sqrt{3} - 40$ —— 1pt.

4. (6 points) Sketch the graph of the function $f(x) = -\log_{1/4}(x+2)+1$, and find its domain, range, x-intercept, y-intercept, and the asymptote(s) as indicated below

a) The graph



(3 pts)

b) The domain= $(-2, \infty)$ —— 1pt.

The range= $(-\infty, \infty)$

c) The x-intercept: $f(x)=0 \Rightarrow \log_{\frac{1}{4}}(x+2)=1 \Rightarrow x+2=\frac{1}{4} \Rightarrow x=\frac{1}{4}-2=-\frac{7}{4}$

$\Rightarrow$ x-int. $(-\frac{7}{4}, 0)$

The y-intercept: $x=0 \Rightarrow f(0) = -\log_{\frac{1}{4}} 2 + 1 = -\log_{\frac{1}{4}}(\frac{1}{4})^{-\frac{1}{2}} + 1 = \frac{1}{2}+1 = \frac{3}{2}$

$\Rightarrow$ y-int. $(0, 3/2)$

—— 1 point

d) The asymptote(s):

$x = -2$, No other asymptotes

—— 1 point

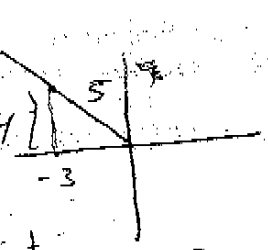
5. (3 points) If $\log 2 = x$ and $\log 3 = y$, then write $\log_5 600$ in terms of x and y.

$\log_5 600 = \frac{\log 600}{\log 5}$ — 1 point

$= \frac{\log 2 + \log 3 + \log 100}{\log 10 - \log 2} = \frac{x + y + 2}{1 - x}$ — 1 point — 1 point

6. (3 points) If the point $(-3,4)$ is on the terminal side of an angle θ in standard position, then find the value of $3\sec\theta - 4\cot\theta$.

$\sec\theta = -\frac{5}{3}$ & $\cot\theta = \frac{-3}{4}$ — 1 point



$3\sec\theta - 4\cot\theta = -5 + 3 = -2$ — 1 point — 1 point

7. (2 points) Find the value of $\sec^2 300° - \cot\frac{3\pi}{4}$.

$\sec 300° = \sec 60° = 2$

$\cot\frac{3\pi}{4} = -\cot\frac{\pi}{4} = -1$ } 1 1/2 points

$\sec^2 300° - \cot\frac{3\pi}{4} = (2)^2 - (-1) = 4 + 1 = 5$ — 1/2 point