King Fahd University of Petroleum and Minerals
College of Sciences
Prep-Year Math Program

KEY

Math 002 Exam I
Term 021 (2002-2003)
Wednesday, October 23, 2002
Time Allowed: 90 Minutes

GRADER

Student's Name: _____

ID #: _____ **Section #:** _____

This exam consists of Two parts

Part I : Multiple Choice Bubble the correct answer on the OMR sheet.
Part II : Written Questions Provide neat and complete solutions.
Show all necessary steps for full credit.

Calculators, Pagers, or Mobiles are NOT allowed during this exam.

Question	Points	Grader
Part I: (1 - 6)	12	
Part II:		
1	4	
2	3	
3	3	
4	4	
5	4	
6	4	
7	3	
8	7	
Total		
	44	

PART – I: (12 Points) Multiple choice questions (MCQ).
Bubble the correct answer on the OMR sheet.

1. The graph of $f(x)=-\left(\frac{2}{3}\right)^x+2$ is the graph of:

(a) an increasing function with y-intercept $(0,1)$.

(b) a decreasing function with y-intercept $(0,1)$.

(c) an increasing function with y-intercept $(0,2)$.

(d) a decreasing function with y-intercept $(0,2)$.

2. The domain D and the range R of the function $y=-\left|\log_{\frac{1}{2}} x^2\right|+1$ are given by:

(a) $D=(-\infty,\ 0)\cup(0,\ \infty)$; $R=(-\infty,1]$.

(b) $D=(-\infty,0)$; $R=(-\infty,\infty)$.

(c) $D=(-\infty,\ 0)\cup(0,\infty)$; $R=[1,\infty)$.

(d) $D=(-\infty,\ 1]$; $R=(-\infty,1)$.

3. If $f(x)=e^{-x}$, then $f(3\ln 2)$ is equal to :

(a) $\frac{1}{8}$.

(b) $\frac{1}{9}$.

(c) $-\frac{2}{3}$.

(d) -6

4. If $\cos 2 = m$, and $\sin 2 = n$, then n – m is:

(a) a positive real number.

(b) a negative real number.

(c) zero.

(d) undefined.

5. If a hard disk in a computer rotates at 2700 revolutions per minute, then the angular speed of the disk in radians per second is equal to:

(a) 90 π radians / second.

(b) 5400 π radians / second.

(c) 3600 π radians / second.

(d) 45 π radians / second.

6. The graph of $y = \frac{3}{4} \cos \frac{x}{2}$, with $-2\pi \le x \le 2\pi$, is below the $x-$axis on the interval:

(a) $(-2\pi, -\pi) \cup (\pi, 2\pi)$.

(b) $(-\pi, \pi)$.

(c) $(-\pi, 0) \cup (\pi, 2\pi)$.

(d) $(-2\pi, 0)$.

PART – II: Written Questions:
[Provide neat and complete solution. Show necessary steps for full credit].

1. (4 points) Write the logarithmic expression

$$5\log_3 x - 8\log_9 y + \log_{\sqrt{3}} z + 1$$

as a single logarithm with a base of 3.

The expression $= \log_3 x^5 - 8\dfrac{\log_3 y}{\log_3 9} + \dfrac{\log_3 z}{\log_3 \sqrt{3}} + \log_3 3$ --- 2 points

$= \log_3 x^5 - 4\log_3 y + 2\log_3 z + \log_3 3$ --- 1 point

$= \log_3 \dfrac{3x^5 z^2}{y^4}$ --- 1 point

2. (3 points) Write $\sec\theta$ in terms of $\cot\theta$, with $\pi < \theta < \frac{3\pi}{2}$.

$\sec^2\theta = 1 + \tan^2\theta = 1 + \dfrac{1}{\cot^2\theta}$ --- 1 point

$\Rightarrow \sec^2\theta = \dfrac{\cot^2\theta + 1}{\cot^2\theta}$ --- 1 point

Thus for $\pi < \theta < \frac{3\pi}{2}$, we get

$\sec\theta = -\dfrac{\sqrt{\cot^2\theta + 1}}{\cot\theta}$ --- 1 point

3. (3 points) Let W be the wrapping function, and let $W(t) = P(x, y)$. Find $W\left(-\frac{45\pi}{4}\right)$.

$W\left(-\frac{45\pi}{4}\right) = \left(\cos -\frac{45\pi}{4}, \sin \frac{-45\pi}{4}\right)$ --- 1 point

But $-\frac{45\pi}{4} = \frac{3\pi}{4} + (-12)\pi$ --- 1 point

$\Rightarrow W\left(-\frac{45\pi}{4}\right) = \left(\cos\frac{3\pi}{4}, \sin\frac{3\pi}{4}\right) = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

--- 1 point

4. (4 points) Solve: $\log_5(x-20)-\log_5\frac{1}{x}=\log 1000$.

$\Rightarrow \log_5 x(x-20) = 3$ ... 1 point

$\Rightarrow x^2 - 20x = 5^3 = 125$ ... 1 point

$\Rightarrow x^2 - 20x - 125 = 0$

$\Rightarrow (x-25)(x+5) = 0$

Thus -5 and 25 are possible solutions ... 1 point

$\Rightarrow$ 25 is the only solution ... 1 point

5. (4 points) Perform the indicated operation and write the result in the simplest form:

$$\frac{1-\cos t}{\sin t} - \frac{1}{\cot t + \csc t}$$

$= \frac{1-\cos t}{\sin t} - \frac{1}{\frac{\cos t}{\sin t} + \frac{1}{\sin t}}$ ... 1 point

$= \frac{1-\cos t}{\sin t} - \frac{\sin t}{\cos t + 1}$ ... 1 point

$= \frac{1-\cos^2 t - \sin^2 t}{\sin t.(\cos t + 1)}$ ... 1 point

$= \frac{0}{\sin t(\cos t + 1)} = 0$ ... 1 point

6. (4 points) Solve: $\frac{10^x - (200)(10^{-x})}{2} = 49$.

$\Rightarrow 10^{2x} - 200 = (98)\,10^x$ --- 1 Point

$\Rightarrow 10^{2x} - (98)\,10^x - 200 = 0$ --- 1 Point

$\Rightarrow (10^x - 100)(10^x + 2) = 0$

$\Rightarrow 10^x = 100$ or $10^x = -2$ --- 1 Point

$\Rightarrow x = 2$ --- 1 point.

7. (3 points) A 22 foot ladder is resting against a wall and makes an angle of 60° with the ground. Find the exact height to which the ladder will reach the wall.

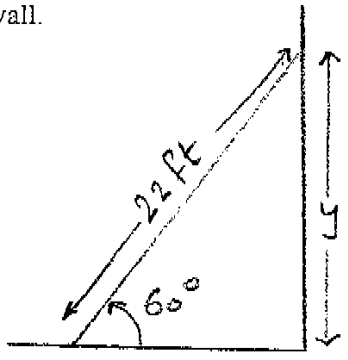
Let y be the required height

$\Rightarrow \sin 60^\circ = \frac{y}{22}$ --- 1 Point

$\Rightarrow y = 22 \sin 60^\circ$

$\Rightarrow y = \frac{22\sqrt{3}}{2} = 11\sqrt{3}$ ft.

--- 1 point.



(1 point)

8. (7 points) Sketch the graph of $y = 5\cot\frac{2x}{3}$ on the interval $[0, 3\pi]$ showing the period, the x – intercepts, and the asymptotes.

The Period: The Period $= \frac{\pi}{\frac{2}{3}} = \frac{3\pi}{2}$ --- 1 point

The x-intercept: $\frac{2x}{3} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

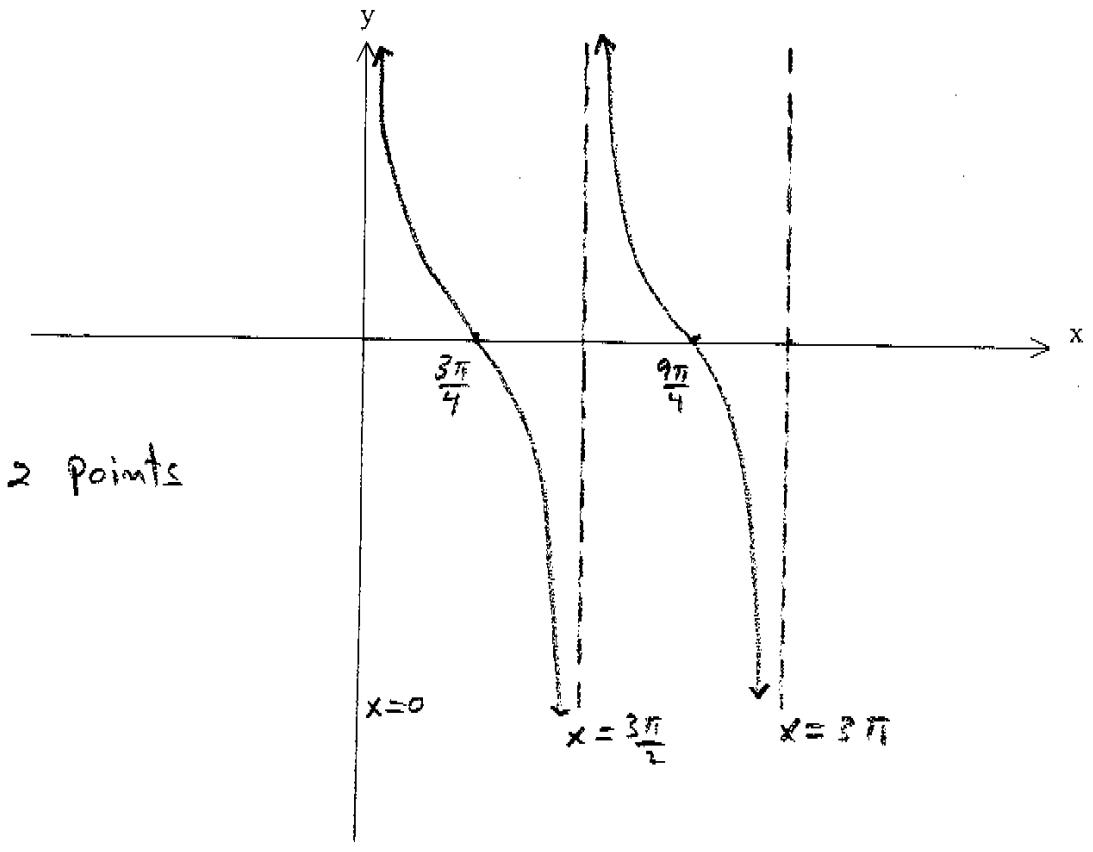
$\Rightarrow$ The x-intercepts are: $\frac{3\pi}{4}$ and $\frac{9\pi}{4}$ on $[0, 3\pi]$ --- 2 points

The asymptotes:

$\frac{2x}{3} = 0, \pi, 2\pi, \ldots$

$\Rightarrow$ The asymptotes are $x = 0$, $x = \frac{3\pi}{2}$, and $x = 3\pi$ on $[0, 3\pi]$ --- 2 points

The graph:



2 Points