King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Semester II, 2005-2006 (052)

MATH 101 – Exam 1

NAME:____	**ID**:_____________________	**Section**: ______

Part 1: Multiple Choice Questions (1 hour)

CODE 001

Question 1 (5 points)

$\lim_{t \to -4} \dfrac{t^2 + 5t + 4}{t^2 + 3t - 4}$ is:

a) $\frac{3}{5}$

b) $\frac{-4}{3}$

c) $\frac{2}{3}$

d) $\frac{-1}{2}$

e) $\frac{2}{5}$

Question 2 (5 points)

Let f be the function defined by $f(x) = \sqrt[4]{9-x}$ and $\varepsilon > 0$ be given. The largest possible δ such that $|f(x) - 0| < \varepsilon$ whenever $-\delta < x - 9 < 0$ is:

a) $\sqrt[4]{9+\varepsilon}$

b) $\sqrt[4]{\varepsilon}$

c) $\sqrt[4]{9-\varepsilon}$

d) ε^4

e) ε^2

Question 3 (5 points)

Let f be the function defined by $f(x)=\begin{cases} 1-x & if \quad x\le 0 \\ \ln x & if \quad 0<x\le 1 \\ x-1 & if \quad 1<x \end{cases}$. Which one of the following statements is correct:

a) f is not continuous from the right at 0 and 1
b) $\lim_{x\to 0} f(x)=\infty$
c) The line $y=0$ is a horizontal asymptote of the curve $y=f(x)$
d) f is discontinuous at 0 and 1
e) f is continuous from the left at 0 and 1

Question 4 (5 points)

$\lim_{x\to -\infty} \dfrac{\sqrt{9x^6-x}}{x^3+2}$ is:

a) 3
b) $+\infty$
c) does not exist
d) $-\infty$
e) -3

Question 5 (5 points)

Let f be the function defined by $f(x)=|x^3|$ for each real number x. The rate of change of f with respect to x at the value c is:

a) does not exist
b) $3c|c|$
c) $3|c^2|$
d) None of these

Question 6 (5 points)

$\lim_{x\to 0^-} \left(\dfrac{1}{x^2+x}-\dfrac{1}{x}\right)$ is:

a) $+\infty$
b) $\dfrac{1}{2}$
c) -1
d) 2
e) $-\infty$

Question 7 (5 points)

Let f be the function defined by $f(x)=\dfrac{2x^2-3x}{|2x-3|}$. Which one of the following statements is true.

a) $\lim\limits_{x\to(\frac{3}{2})^-} f(x)=\dfrac{3}{2}$

b) $\lim\limits_{x\to\frac{3}{2}} f(x)=\dfrac{3}{2}$

c) $\lim\limits_{x\to(\frac{3}{2})^+} f(x)$ does not exist

d) $\lim\limits_{x\to\frac{3}{2}} f(x)$ does not exist and $\lim\limits_{x\to(\frac{3}{2})^+} f(x)=\dfrac{3}{2}$

e) $\lim\limits_{x\to\frac{3}{2}} f(x)$ does not exist and $\lim\limits_{x\to(\frac{3}{2})^-} f(x)=\dfrac{3}{2}$

Question 8 (5 points)

The constants a and b that make the function $f(x)=\begin{cases} a\dfrac{\sin x}{x} & if \quad -1\le x<0 \\ 3x^2-3x+2 & if \quad 0\le x\le 1 \\ \dfrac{x^2-1}{x-b} & if \quad 1<x\le 3 \end{cases}$

satisfy the conditions of the intermediate value theorem on [-1,3] are:

a) $a=0$ and $b=2$
b) $a=2$ and $b=3$
c) $a=1$ and $b=1$
d) $a=1$ and $b=3$
e) $a=2$ and $b=1$

Question 9 (5 points)

Let f be the function defined by $f(x)=\sqrt{x^2+x+1}+x$. Then $\lim\limits_{x\to-\infty} f(x)$ is

a) $\dfrac{1}{2}$

b) $\dfrac{-1}{2}$

c) $-\infty$

d) $+\infty$

e) does not exist

Question 10 (5 points)
A rough sketch of the derivative of the following function

Would be

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Part 2: Essay Questions (1 hour)

	Score (out of 10)
Question 1	
Question 2	
Question 3	
Question 4	
Question 5	
Total (out of 50)	

Question 1

Use the squeezing theorem to find $\lim_{x \to 0} x^2 e^{\sin(\frac{1}{x})}$.

Question 2

Let $\lim_{x \to 3} f(x) = 0$ and $\lim_{x \to 3} h(x) = 5$. Use these limits and the given graph of the function g to evaluate each of the following limits if it exists. If the limit does not exist, explain why.

a) $\lim_{x \to 3} (f(x) - \frac{h(x)}{3})$

b) $\lim_{x \to 3} \frac{g(x) - 3}{h(x)}$

c) $\lim_{x \to 3^-} (g(x) + h(x))^3$

d) $\lim_{x \to 0^+} \frac{1}{\sqrt{g(x)}}$

e) $\lim_{x \to 3} f(x)g(x)$

Question 3

By using the ε and δ definition, prove that $\lim_{x \to 4} \dfrac{1}{x-2} = \dfrac{1}{2}$.

Question 4

Prove that the equation $x^2 + x - \cos x = 0$ has at least two solutions in the interval $[-\dfrac{\pi}{2}, \dfrac{\pi}{2}]$.

Question 5

Suppose that f is a continuous function on the interval $[0,1]$ and $f(0) = f(1)$. Prove (analytically and not geometrically) that there exists $a \in (0, \frac{1}{2})$ such that a and $a + \frac{1}{2}$ have the same image, that is, $f(a) = f(a + \frac{1}{2})$.

(**Hint**: consider the function $g(x) = f(x + \frac{1}{2}) - f(x)$)