King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Math 345-01 The First Exam
Tuesday November 07, 2006
Dr. Abdulaziz Al-Assaf
Time Allowed: 100 Minutes

Name:____

Show all your work.

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1. Prove that $H = \{2^n 6^m : m, n \in Z\}$ is a group under multiplication.

2. Let G be a group and H be a nonempty subset of G. Prove that H is a subgroup of G if $ab^{-1} \in H$ whenever $a, b \in H$.

3. Find the center of $Gl(2, R)$.

4. Suppose that $G =< a >$ and $| a |= 20$. Find all the subgroups of G.

5. Let $G =< a >$ be a cyclic group of order n. Prove that $G =< a^k >$ if and only if $\gcd(k, n) = 1$..

6. Prove or disprove:

a. If $\phi : G \to \breve{G}$ is an isomorphism and $K \leq G$, then $\phi(K) \leq \breve{G}$ where $\phi(K) = \{\phi(k) \mid k \in K\}$.

b. $U(10) \approxeq U(12)$.

7. Let $H = \{\alpha \in S_n \mid \alpha(1) = n, \alpha(n) = 1\} \bigcup \{\alpha \in S_n \mid \alpha(1) = 1, \alpha(n) = n\}$ and let $n > 1$.

a. Show that H is a subgroup of S_n.

b. Find the order of H.

c. Find the order of the permutation $\begin{pmatrix} 123456789 \\ 314259687 \end{pmatrix}$.

8. Find the number of all elements of order 15 in the group $Z_{45} \oplus Z_{25}$.

a. State and prove Fermat's Little Theorem

b. Suppose that K is a proper subgroup of H and H is a proper subgroup of G. If the order of K is 42 and the order of G is 420, what are the possible orders of H.