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Some Models for the Voltage Induced in a Liquid by Cavitation

Alina Barbulescu and Vasile Marza
"Ovidius" University of Constanţa,
Bd. Mamaia 124, 8700, Constanţa, Romania
E-mail: abarbulescu@univ-ovidius.ro, vmarza@univ-ovidius.ro

Abstract

An ultrasonic field that goes over a liquid can produce or move cavitation bubbles and these ones determine physical and chemical effects. Experimentally it was proved that an alternative voltage appears and it contains both the ultrasonic frequency and the inherent harmonics and subharmonics. This conclusion results also by the Fourier analysis made for the intercepted signal.

The aim of this paper is to determine a mathematical relation for the voltage induced by the ultrasonic cavitation at the exterior of the cavitation bubbles.

Also, using a model of this type, the logarithmic decrement for the liquids, particularly for the water, can be deduced.

1 Introduction

When an acoustic wave propagates through a liquid containing microscopic gas inclusions, these "nucleation sites" can be mechanically activated, at which point they spawn free bubbles which then undergo highly energetic volume pulsations.

In a liquid, an ultrasonic field can carry along small bubbles or can produce cavitation bubbles, whose moments determine drastic effects as: erosion, unpassivation and emulsification, chemical reactions, sonoluminescence, pressure variation (characterized by oscillations with frequencies different from that of the ultrasound which generates the bubbles).

2 Method

In order to determine the voltage appearing in the ultrasonic cavitation, the experimental set-up given in Fig. 1 was used ([1]). It is composed, essentially, by a high frequency generator, a transducer — core tank, an active network transformer and power transistors.

The transducer — core tank is formed by two transducers, put on the bottom of the core tank.

The high frequency generator is of autodyne type. It is realized with power commutation transducers and with current supply from the network doughnut.

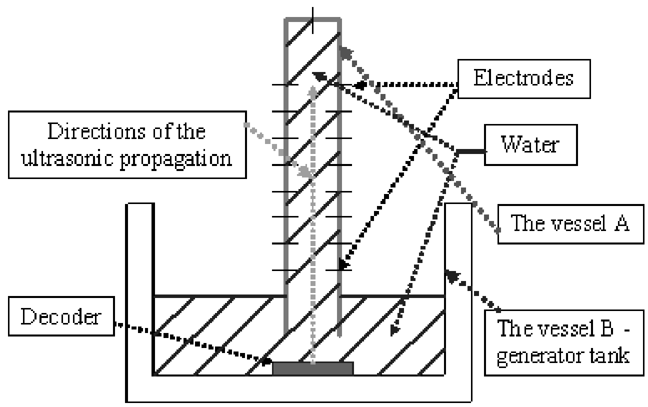

Figure 1: The experimental set-up

The vessel A, which contains the measurement electrodes, was introduced in the vessel B, which is the ultrasound generator tank. The study was made at 80W (the first power level), 120W (the second power level), 180W (the third power level of the ultrasonic generator). The studied liquid, contained in the vessels A and B, was the water.

The acquisition card can be connected to different measurement electrodes, which can lie at different distances. The acquisition card captures the electric signals, which are processed by a computer.

The activity of the ultrasonic unit is based on the principle of the acoustic cavitation due to the ultrasonic waves and can work at 3 power degrees.

3 Results

The voltage induced by the ultrasonic field depends on the liquid nature, the ultrasonic frequency, the distance between the electrodes and the power of the electric generator. It was deduced that the calculus relation for the induced voltage is ([2]):

$$U = k\frac{neaD}{\alpha N_0},$$

where:

U is the induced voltage,
N_0 — Avogadro's number,
a — the atomic interspace,
D — the distance between the electrodes,
e — the elementary electrical charge,
n — the electrons number which lie inside a cavitation bubble of a given radius.

In order to determine a new mathematical model for the induced voltage, Box-Jenkins method will be used. The following definitions are known ([3, 4]).

Definition 1 A *time series* is a realization of a stochastic process or a sequence of values that shows the volume variation of a statistic population or of the characteristic's level, related to the time.

Definition 2 A *discrete time process* is a sequence of random variables $X_t;\ t \in \mathbb{Z}$.

Definition 3 A discrete time process $(X_t;\ t \in \mathbb{Z})$ is called *stationary* if:

$$(\forall) t \in \mathbb{Z},\ M(X_t^2) < \infty,$$
$$(\forall) t \in \mathbb{Z},\ M(X_t) = \mu,$$
$$(\forall) t \in \mathbb{Z},\ (\forall) h \in \mathbb{Z},\ \mathrm{Cov}(X_t,\ X_{t+h}) = \gamma(h),$$

where $M(X)$ is the expectation value of X and $\mathrm{Cov}(X, Y)$ is the correlation of the variables X and Y.

Definition 4 A stationary process $(\varepsilon_t;\ t \in \mathbb{Z})$ is called a *white noise* if $\gamma(h) = 0$ for $h \neq 0$, $M(\varepsilon_t) = 0$ and $D^2(\varepsilon_t) = \sigma^2 = \gamma(0)$, $(\forall) t \in \mathbb{Z}$.

Definition 5 The function defined on $\mathbb{Z}$ by

$$\rho(h) = \frac{\mathrm{Cov}(X_t, X_{t+h})}{\sqrt{D^2(X_t) D^2(X_{t+h})}} = \frac{\gamma(h)}{\gamma(0)}$$

is called the *autocorrelation function* (ACF) of the process $(X_t;\ t \in \mathbb{Z})$.

Definition 6 If $(X_t;\ t \in \mathbb{Z})$ is a stationary process, the function defined by

$$\tau(h) = \frac{\mathrm{Cov}(X_t - X_t^*, X_{t-h} - X_{t-h}^*)}{D^2(X_t - X_t^*)}, \quad h \in \mathbb{Z}_+,$$

is called the *partial autocorrelation function* (PACF), where X_t^* (X_{t-h}^*) is the affine regression of X_t (X_{t-h}) with respect to $X_{t-1}, ..., X_{t-h+1}$.

Definition 7 Consider

$$\begin{aligned} B(X_t) &= X_{t-1}\,, \\ \Phi(B) &= 1-\varphi_1 B-\cdots-\varphi_p B^p,\ \varphi_p\neq 0, \\ \Theta(B) &= 1-\theta_1 B-\theta_2 B^2-\cdots-\theta_p B^q,\ \theta_p\neq 0, \\ \Delta^d X_t &= (1-B)^d X_t\,. \end{aligned}$$

The process $(X_t;\ t\in\mathbb{Z}_+)$ is called ARIMA(p,d,q) if:

$$\Phi(B)\Delta^d X_t=\Theta(B)\varepsilon_t,$$

where the absolute values of the roots of Φ and Θ are greater than 1 and $(\varepsilon_t,\, t\in\mathbb{Z})$ is a white noise.

If $d=0=q$, the ARIMA(p,d,q) process is an AR(p) process.
If $d=0=p$, the ARIMA(p,d,q) process is an MA(q) process.
If $d=0$, the ARIMA(p,d,q) process is an ARMA(p,q) process.

Remark 1 The autocorrelation function of an AR(p) (autoregressive of p-th order) process is an exponential decreasing or a damped sine wave oscillation. The partial autocorrelation function of an AR(p) process is vanishing if $h>p$ and $\tau(p)=\varphi_p$.

Remark 2 The autocorrelation function of an MA(q) (moving average of q-th order) process is vanishing for $h>q$. The partial autocorrelation function of an MA(q) process is nonvanishing starting at some lag value.

Remark 3 For an ARMA(p,q) (autoregressive moving average of p-th and q-th orders) process, the autocorrelation function is a mixture of exponential decreasing curves and damped sine wave oscillation, when $p>q$ and when $p<q$, the autocorrelation function is of the previous type if $h>q-p$.

The data obtained (the voltage induced by the cavitation bubbles), using the acquisition card, at power equal to 180W are represented in Fig. 2. The horizontal axis is the time axis (in 0.000001s) and the vertical axis is the voltage axis (in V).

It can be seen that there exists a periodicity of the voltage and after the study of the given values, the determined period was 0.000101 s. So, the study was made only for a period.

First, the autocorrelation function (ACF) of the voltage was studied for the lags between 1 and 16 and it was determined the confidence interval at the confidence level 95%.

In Fig. 3 it can be seen that there are values of ACF outside the confidence interval.

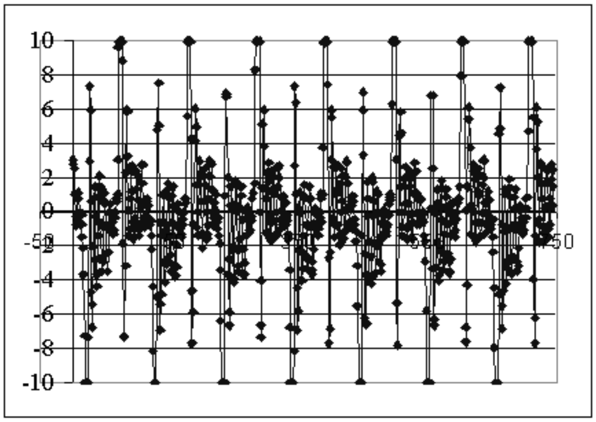

Figure 2: The signal form

The form of the ACF is of damped sine wave oscillation and also is that of PACF (Figure 4). These forms enable us to think that the process could be of ARIMA type.

The study was made for three models, respectively of ARIMA(2,1,2), ARIMA(2,1,1), ARIMA(2,1,0) type. To choose between them, the test Schwarz (SBC) and Akäike (AIC) were used. The preferable values are that of the SBC test. The corresponding values of SBC and the AIC are, respectively: 316.7732 (327.1939), 317.8674 (325.8829), 317.1772 (322.3876).

The Student test made for the coefficients of each model shows that a coefficient in the first model is zero. Therefore, the model chosen is:

$$(1 - 1.2313304B + 0.84409B^2)(1 - B)V_n = \varepsilon_n,$$

where $n \in \mathbb{N}$, $n \geq 3$ and $\{\varepsilon_n,\ n \in \mathbb{N}\}$ is the residual.

The model chosen is of ARIMA(2,1,0) type, without a constant term. Also the model ARIMA(2,1,0) with a constant term was studied, but the significance test made for the coefficients led us to reject the hypothesis that the constant was non-vanishing.

In order to prove that the model is a good one, the autocorrelation function and the partial autocorrelation function of the residuals were studied. The graphs of these functions can be seen on Figures 5 and 6.

The values of the autocorrelation function and of the partial autocorrelation function of the residuals are inside the confidence intervals, at 0.95 confidence level.

The values of the Box-Ljung statistics are in the interval $[0.493, 6.063]$, so they are less than $\chi^2(102)$.

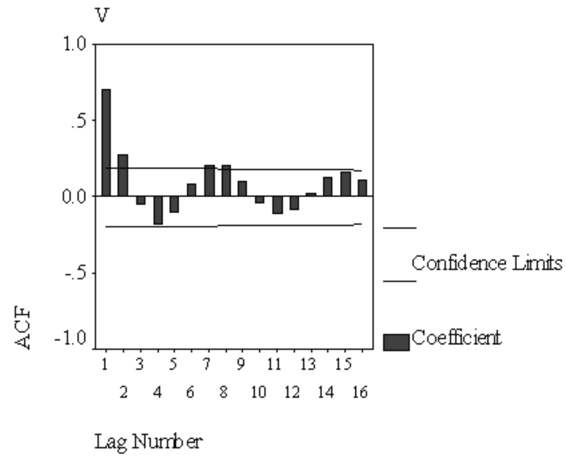

Figure 3: The ACF of the voltage, V

The probabilities to accept the hypothesis that the residuals form a white noise are around 0.90%.

Therefore, the residuals form a white noise and the model is well selected.

4 Conclusions

In our experiments it was found that when an ultrasound propagates through a liquid, a potential difference between two points appears. It is induced by the cavitation bubbles, is almost-periodic and has both harmonic and subharmonic components.

The voltage depends also on the distance between the electrodes, the power degree of the ultrasound generator, the liquid, the radius of the cavitation bubbles.

The mathematical model found by us differs from that obtained at 80W, which was ([5]) an AR(2), given by:

$$V_n = 1.5636298V_{n-1} - 0.89193194V_{n-2} + \varepsilon_n,$$

where $n \geq 3$ and $\{\varepsilon_n,\ n \in \mathbb{N}\}$ is a white noise.

It differs also from that obtained at 120W, which was of ARMA(2,1) type, given by ([6]):

$$V_n = 1.3006553V_{n-1} - 0.7035790V_{n-2} + \varepsilon_n - 0.6128040\varepsilon_{n-1}\,,$$

where $n \in \mathbb{N}$, $n \geq 3$ and $\{\varepsilon_n,\ n \in \mathbb{N}\}$ is the residual.

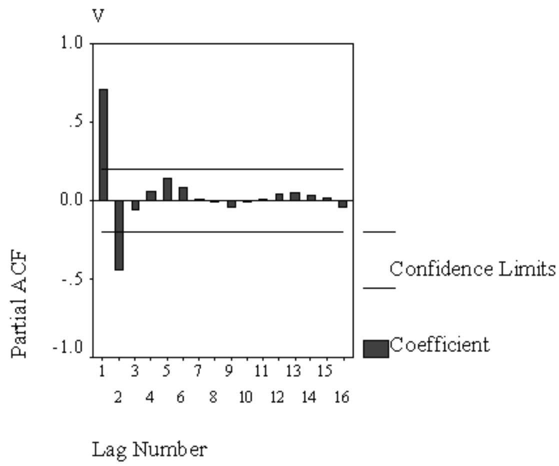

Figure 4: The PACF of the voltage, V

The study made can be used to determine the acoustic logarithmic decrement for the water ([1]). The result obtained coincides with that known from the literature.

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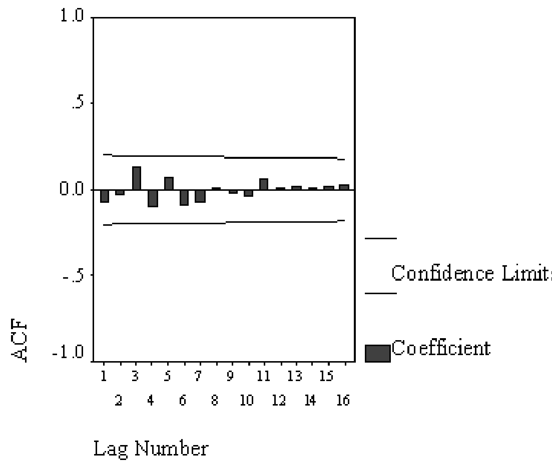

Figure 5: The ACF of the residuals in ARIMA(2,1,0)

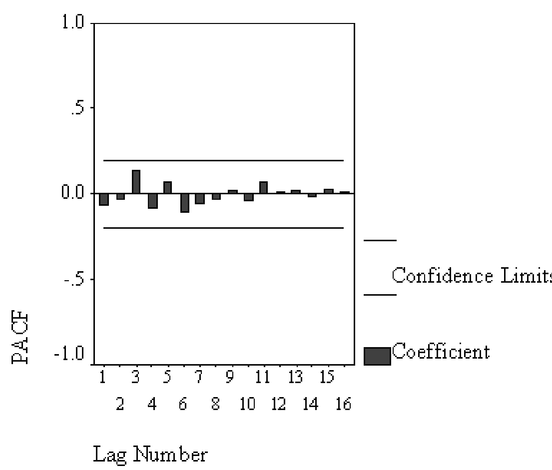

Figure 6: The PACF for the residuals in ARIMA(2,1,0)