King Fahd University of Petroleum and Minerals
Department of Mathematics
Math 201 Sem I 2004

Second Major Exam Sat 4 / 12 / 2004 Time $1\frac{1}{4}$ hours

Name: Key I.D.#: ______________ Serial #: ___

Section #: 13 16

Answer all the questions

Show all of your work

Question #	1	2	3	4	5	6	7	Total
Grade	/5	/6	/5	/5	/5	/6	/8	/40

1. Use total differential to approximate the change in $f(x,y) = x^2 + 3xy - 2y^2 - 4y$ as (x,y) varies from $P(2,-1)$ to $Q(1.98,-0.99)$.

$\Delta x = 1.98 - 2 = -0.02$, $\Delta y = -0.99 - (-1) = 0.01$

$df = f_x \Delta x + f_y \Delta y$

$f_x = 2x + 3y \Big|_{(2,-1)} = 4 - 3 = 1$, $f_y = 3x - 4y - 4 \Big|_{(2,-1)}$
$= 6 + 4 - 4 = 6$

$df = 1(-0.02) + 6(0.01) = -0.02 + 0.06$

$= 0.04$ is the change in the value of the function

Math 201 T2 Key

2. Find equation of the plane that contains the point $P(1,-1,0)$ and the line of intersection of the two planes $2x+y-2z+1=0$ and $4x-y+z+3=0$

$\vec{n}_1 = \langle 2, 1, -2 \rangle$, $\vec{n}_2 = \langle 4, -1, 1 \rangle$

$$\vec{V} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} i & j & k \\ 2 & 1 & -2 \\ 4 & -1 & 1 \end{vmatrix} = i(1-2) - j(2+8) + k(-2-4)$$
$$= -\vec{i} - 10\vec{j} - 6\vec{k}$$

A point on the line: $x=0$:
$$y - 2z + 1 = 0$$
$$-y + z + 3 = 0$$
$$-z + 4 = 0 \Rightarrow z = 4$$
so $y = 7$

$Q(0,7,4)$. $\vec{PQ} = \langle -1, 8, 4 \rangle$

$$\vec{n} = \vec{PQ} \times \vec{V} = \begin{vmatrix} i & j & k \\ -1 & 8 & 4 \\ -1 & -10 & -6 \end{vmatrix} = i(-48+40) - j(6+4) + k(10+8)$$
$$= -8\vec{i} - 10\vec{j} + 18k$$ is the normal vector

$$-8(x-1) - 10(y+1) + 18(z) = 0$$
$$-8x - 10y + 18z - 2 = 0$$

3. Determine whether the limit exist or not (show all details) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^3}{2x^2+y^6}$

Consider the two paths approach:-

Along x-axis, $y=0$,

$$\lim_{(x,0)\to(0,0)} \frac{0}{2x^2+0} = 0$$

Along the curve $x=y^3$:

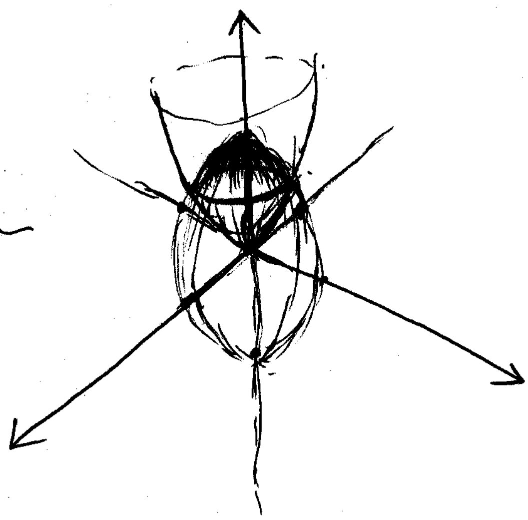
$$\lim_{(x,x^3)\to(0,0)} \frac{y^6}{2y^6+y^6} = \frac{1}{3}$$, so the limit does n

Math 201 Test 2 Key

4. **Sketch the region enclosed by the paraboloid $z = x^2 + y^2$, and the ellipsoid $2x^2 + 2y^2 + z^2 = 4$, and describe their curve of intersection .**

$z = x^2 + y^2$

$\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{4} = 1$



The curve of intersection is:

$2z + z^2 - 4 = 0$

$z^2 + 2z + 1 - 5 = 0$

$(z+1)^2 = 5 \implies z + 1 = \pm\sqrt{5}$, $z = -1 + \sqrt{5}$ o.k.

$z = -1 - \sqrt{5}$ ignored

$\cancel{x^4 + y^4 + 1 + 2x^2y^2 + 2x^2 + 2y^2 = 5}$

$x^2 + y^2 = \sqrt{5} - 1$ which is a circle centered at the origin with radius $\sqrt{\sqrt{5} - 1}$

5. **Find the equation of the sphere that is centered at the point $P(0, 1, -2)$ and tangent to the plane $x - 2y + 2z - 1 = 0$**

Radius of the sphere = Distance from p to the plane

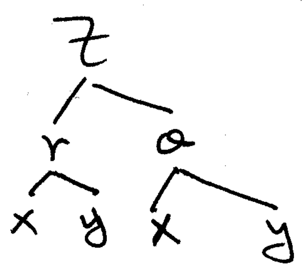
$$D = \frac{|0 - 2 - 4 - 1|}{\sqrt{1 + 4 + 4}} = \frac{7}{3}$$

So the equation of the sphere is:

$$x^2 + (y-1)^2 + (z+2)^2 = \frac{49}{9}$$

Math 201 T2 key

6. Let $z = f(x,y)$ is expressed in the polar form $z = g(r,\theta)$ by making the substitution $x = r\cos\theta$ and $y = r\sin\theta$. View r and θ as functions of x and y, and use implicit differentiation to show that $\frac{\partial r}{\partial x} = \cos\theta$ and $\frac{\partial \theta}{\partial x} = -\frac{\sin\theta}{r}$.



$x = r\cos\theta$

$y = r\sin\theta$

Use implicit diff. by x, :

$$1 = \frac{\partial r}{\partial x}\cos\theta - r\sin\theta\cdot\frac{\partial\theta}{\partial x} \quad \text{--- (1)}$$

$$0 = \frac{\partial r}{\partial x}\sin\theta + r\cos\theta\cdot\frac{\partial\theta}{\partial x} \quad \text{--- (2)} \Rightarrow r\frac{\partial\theta}{\partial x} = -\frac{\frac{\partial r}{\partial x}\sin\theta}{\cos\theta} \quad \text{--- (3)}$$

$$1 = \frac{\partial r}{\partial x}\cos\theta + \frac{\sin\theta\cdot\sin\theta\cdot\frac{\partial r}{\partial x}}{\cos\theta} = \frac{\frac{\partial r}{\partial x}(\cos^2\theta+\sin^2\theta)}{\cos\theta}$$

$$= \frac{\frac{\partial r}{\partial x}}{\cos\theta} \Rightarrow \frac{\partial r}{\partial x} = \cos\theta. \quad \text{--- (4)}$$

substitute (4) in (3)

$$r\frac{\partial\theta}{\partial x} = \frac{-\cancel{\cos\theta}\cdot\sin\theta}{\cancel{\cos\theta}} \Rightarrow \frac{\partial\theta}{\partial x} = -\frac{\sin\theta}{r} \quad \sim (5)$$

So the required.

2nd solution

$$r = \sqrt{x^2+y^2} \Rightarrow \frac{\partial r}{\partial x} = \frac{2x}{2\sqrt{x^2+y^2}} = \frac{2r\cos\theta}{2r} = \cos\theta \checkmark$$

$$\tan\theta = \frac{y}{x} \Rightarrow \sec^2\theta\cdot\frac{\partial\theta}{\partial x} = \frac{-y}{x^2} \Rightarrow \frac{\partial\theta}{\partial x} = \frac{-y}{x^2\sec^2\theta} = \frac{-r\sin\theta}{r^2\cos^2\theta\sec^2\theta} = -\frac{\sin\theta}{r} \checkmark$$

7. For each of the following give a short answer in the assigned space:

a. The normal vector to the plane $3x + 7y - z + 2 = 0$ is equal to

$n = \langle 3, 7, -1 \rangle$

b. The graph of the spherical equation $\rho = 4\cos\phi$ has an equivalent rectangular equation equals to

$x^2+y^2+z^2-4z=0$ | $\rho^2 = 4\rho\cos\phi$

c. The level surface (equation) for $f(x,y,z) = x^2 + 4y^2 - z^2$ that passes through the point $P(2,1,1)$ is

$$f(2,1,1) = 4+4-1 = 7$$

$x^2+4y^2-z^2=7$

d. The quadric surface $9x^2 + y^2 - 2z^2 + 4z + 3 = 0$ is called

$$9x^2+y^2-2(z^2-2z+1)+3 \overset{+2}{=} 0$$
$$9x^2+y^2-2(z-1)^2+5=0$$
$$9x^2+y^2-2(z-1)^2=-5$$
$$-\frac{9x^2}{5}-\frac{y^2}{5}+\frac{2}{5}(z-1)^2=1$$

Hyperboloid of Two sheets

e. Let $f(x,y,z) = \ln(x^2+2y^2+4z^2)$, then find $\frac{\partial f(1,2,3)}{\partial z}$

$$f_z = \frac{8z}{x^2+2y^2+4z^2}\Bigg|_{(1,2,3)} = \frac{24}{1+8+36} = \frac{24}{45} = \frac{8}{15}$$

$\frac{8}{15}$

f. Let the point $P(2, \frac{\pi}{6}, 1)$ with cylindrical coordinates then the equivalent

$r=2,\quad x = 2\cos\frac{\pi}{6} = 2\cdot\frac{\sqrt{3}}{2} = \sqrt{3}$

$y = 2\sin\frac{\pi}{6} = 2\cdot\frac{1}{2} = 1$

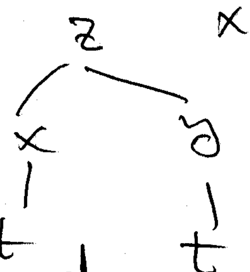
rectangular coordinates are $(\sqrt{3},1,1)$

$$\rho = \sqrt{4+1} = \sqrt{5}$$
$$\tan\phi = \frac{2}{1} = 2$$
$$\phi = \tan^{-1}2$$

and spherical coordinates are $(\sqrt{5}, \frac{\pi}{6}, \tan^{-1}2)$

g. If $z = f(x,y) = 2x^2 + 3xy$, and $x = 2t$, $y = \frac{1}{t}$, then find $\frac{dz}{dt}$ at $t = 1$

$x=2,\ y=1$



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$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial z}{\partial y}\cdot\frac{dy}{dt}$$
$$= (4x+3y)\cdot 2 + 3x\cdot\frac{-1}{t^2}$$
$$= 11\cdot 2 + 6\cdot(-1) = 22-6$$

[redacted]