Math 201 - 1 Quiz # 3 A Sem 053

Name: Key I.D.#:______________ Serial #:_____

Q1 Find equation of the plane that passes through the point $P(-1,2,0)$ and contains the line $L: x=-2t+1 \quad ,y=t+3 \quad ,z=2t$, $\vec{v} = \langle -2,1,2 \rangle$

Let $Q(1,3,0)$ be a point on the line

$\vec{PQ} = \langle 2,1,0 \rangle$, $\vec{n} = \vec{v} \times \vec{PQ} = \begin{vmatrix} i & j & k \\ -2 & 1 & 2 \\ 2 & 1 & 0 \end{vmatrix} = \vec{i}(-2) - \vec{j}(-4) + \vec{k}(-4)$

$= -2\vec{i} + 4\vec{j} - 4\vec{k}$

So The equation of the plane: $-2(x+1)+4(y-2)-4(z-0)=0$

$-2x+4y-4z-10=0$

Q2 Identify and sketch the surface $z^2 = x^2+4y^2+2x-8y+1$

complete square in x & y, $z^2 = x^2+2x+1+4(y^2-2y+1)+1-1-4$

$z^2 = (x+1)^2+4(y-1)^2-4 \Rightarrow (x+1)^2+4(y-1)^2-z^2=4$

$\frac{(x^2+1)^2}{4} + (y-1)^2 - \frac{z^2}{4} = 1$ Hyperboloid of one sheet (elliptic)

- extended in the z-direction
- Cross section is ellips

Q3 Let $Q(1,-1,\sqrt{2})$ be a point with rectangular coordinates , give equivalent
a. cylindrical coordinates and b. spherical coordinates

$r = \sqrt{1+1} = \sqrt{2}$, $\tan\theta = \frac{-1}{1} = -1 \Rightarrow \theta = \frac{-\pi}{4}$

a. cylindrical coord. are $(\sqrt{2}, \frac{-\pi}{4}, \sqrt{2})$

b. $\rho = \sqrt{1+1+2} = 2$, $\cos\phi = \frac{z}{\rho} = \frac{\sqrt{2}}{2} \Rightarrow \phi = \frac{\pi}{4}$

so spherical coord. are $(2, \frac{-\pi}{4}, \frac{\pi}{4})$.

Math 201 - 1 Quiz # 3 B Sem 053

Name: Key I.D.#:________________ Serial #:_____

Q1 Find equation of the plane that passes through the point $P(2,-1,3)$ and contains the line $L: x = t-1 \quad , y = 2t+1 \quad , z = -2t-2$ $\quad \vec{v} = \langle 1, 2, -2 \rangle$

let $Q(-1,1,-2)$ be a point on the line, $\vec{PQ} = \langle -3, 2, -5 \rangle$

$$\vec{n} = \vec{PQ} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & 2 & -5 \\ 1 & 2 & -2 \end{vmatrix} = \vec{i}(-4+10) - \vec{j}(6+5) + \vec{k}(-6-2)$$
$$= 6\vec{i} - 11\vec{j} - 8\vec{k}$$

So the equation of the plane: $6(x-2) - 11(y+1) - 8(z-3) = 0$

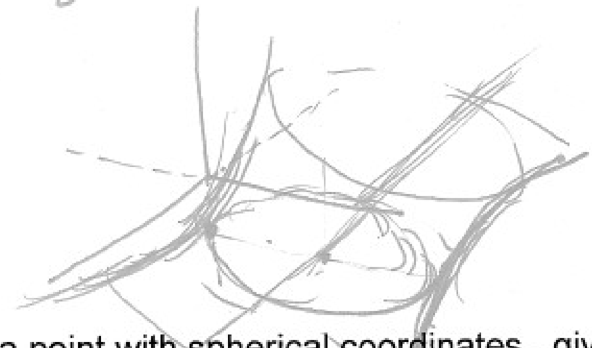
$6x - 11y - 8z + 1 = 0$

Q2 Identify and sketch the surface $x^2 = 4z^2 + y^2 + 8z - 4y$

$4(z^2+2z+1) + y^2 - 4y + 4 - x^2 = 8$

$4(z+1)^2 + (y-2)^2 - x^2 = 8$

$\frac{(z+1)^2}{2} + \frac{(y-2)^2}{8} - \frac{x^2}{8} = 1$ Hyperboloid of one sheet



Q3 Let $Q(3, \frac{\pi}{6}, \frac{\pi}{3})$ be a point with spherical coordinates , give equivalent

a. cylindrical coordinates and b. rectangular coordinates

a. $r = \rho \sin\phi = 3 \cdot \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$, $z = \rho\cos\phi = 3 \cdot \frac{1}{2} = \frac{3}{2}$

$\Rightarrow$ The cylindrical coordinates $\left(\frac{3\sqrt{3}}{2}, \frac{\pi}{6}, \frac{3}{2}\right)$

b. $x = r\cos\theta = \frac{3\sqrt{3}}{2} \frac{\sqrt{3}}{2} = \frac{9}{4}$, $y = r\sin\theta = \frac{3\sqrt{3}}{2} \cdot \frac{1}{2} = \frac{3\sqrt{3}}{4}$

$\Rightarrow$ The rectangular coordinates $\left(\frac{9}{4}, \frac{3\sqrt{3}}{4}, \frac{3}{2}\right)$

Math 201 - 4 Quiz # 3 Sem 053

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Q1 Find equation of the plane that contains the two lines

$L_1 : x = t-1 \quad ,y = 2t+1 \quad ,z = -2t-2 \qquad L_2 : x = -2t+1 \quad ,y = -4t-1 \quad ,z = 4t$

$\vec{v}_1 = \langle 1, 2, -2 \rangle$, $P_1(-1, 1, -2)$, $P_2(1, -1, 0)$

$\overrightarrow{P_1P_2} = \langle 2, -2, 2 \rangle = 2\underline{\langle 1, -1, 1 \rangle} = 2\vec{v}_2$

$$\vec{v}_1 \times \vec{v}_2 = \vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & -2 \\ 1 & -1 & 1 \end{vmatrix} = \vec{i}(2-2) - \vec{j}(1+2) + \vec{k}(-1-2)$$

$$= -3\vec{j} - 3\vec{k} = -3(\vec{j} + \vec{k})$$

$(y-1) + (z+2) = 0$

$y + z + 1 = 0$

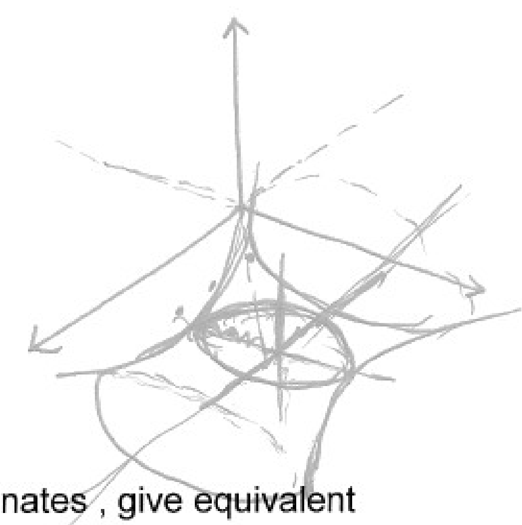
Q2 Identify and sketch the surface $x^2 = 4z^2 + y^2 + 8z - 4y + 4x$

$4(z^2 + 2z + 1) + y^2 - 4y + 4 - (x^2 - 4x + 4) = 4$

$4(z+1)^2 + (y-2)^2 - (x-2)^2 = 4$

$(z+1)^2 + \dfrac{(y-2)^2}{4} - \dfrac{(x-2)^2}{4} = 1$

Hyperboloid of one sheet



Q3 Let $Q(2, \frac{\pi}{4}, \frac{\pi}{6})$ be a point with spherical coordinates, give equivalent

a. cylindrical coordinates and b. rectangular coordinates

$\rho = 2, \quad \theta = \frac{\pi}{4}, \quad \phi = \frac{\pi}{6}$

a. $r = \rho \sin\frac{\pi}{6} = 2 \cdot \frac{1}{2} = 1$, $\quad z = \rho \cos\frac{\pi}{6} = 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}$

$(1, \frac{\pi}{4}, \sqrt{3})$ is cylindrical coordinates

b. $x = r\cos\theta = 1 \cdot \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$, $\quad y = 1 \sin\theta = \frac{1}{\sqrt{2}}$

$(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \sqrt{3})$ is rectangular coordinates