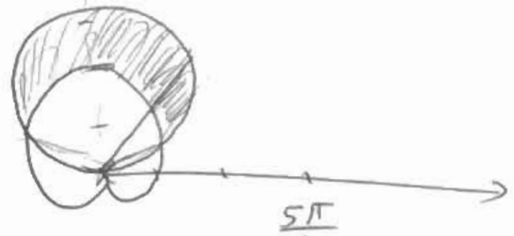
Math 201 - 1 Quiz # 2 B Sem 053

Name: Key I.D.#:__________ Serial #:_____

Q1 Find the area inside the polar curve $r = 3\sin\theta$ and outside the curve $r = 1 + \sin\theta$

$3\sin\theta = 1 + \sin\theta \Rightarrow \sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$



$$A = \frac{1}{2}\int_{\pi/6}^{5\pi/6}\left[(3\sin\theta)^2 - (1+\sin\theta)^2\right]d\theta = \frac{1}{2}\int_{\pi/6}^{5\pi/6}\left[9\sin^2\theta - 1 - 2\sin\theta - \sin^2\theta\right]d\theta$$

$$= \int_{\pi/6}^{\pi/2}\left[4 - 4\cos 2\theta - 1 - 2\sin\theta\right]d\theta =$$

$$= \left[3\theta - 2\sin 2\theta + 2\cos\theta\right]_{\pi/6}^{\pi/2} = \underbrace{3\left(\frac{\pi}{2} - \frac{\pi}{6}\right)}_{=\pi} - 2\left(0 - \frac{\sqrt{3}}{2}\right) + 2\left(0 - \frac{\sqrt{3}}{2}\right)$$

Q2 Find equation of the sphere that is centered at $C(0,3,-1)$ and tangent from outside to the sphere $x^2 + y^2 + z^2 - 4x - 4y - 2z + 5 = 0$

$x^2 - 4x + 4 + y^2 - 4y + 4 + z^2 - 2z + 1 = -5 + 4 + 4 + 1$

$(x-2)^2 + (y-2)^2 + (z-1)^2 = 4$, $C_1(2,2,1)$, $r = 2$

$d(c_1, c) = \sqrt{2^2 + 1 + 2^2} = 3$. $\Rightarrow$ Radius of the required circle is 1

$\therefore$ The equation is:

$x^2 + (y-3)^2 + (z+1)^2 = 1$

Q3 Let the vectors $\mathbf{u} = \langle 1,-2,0\rangle$, and $\mathbf{v} = \langle 1,2,1\rangle$, find a unit vector parallel to the curve $3\mathbf{v} - \mathbf{u}$

, $3\vec{v} - \vec{u} = 3\langle 1,2,1\rangle - \langle 1,-2,0\rangle = \langle 2,8,3\rangle$

a unit vector $= \dfrac{3\vec{v} - \vec{u}}{\|3\vec{v} - \vec{u}\|} = \dfrac{\langle 2,8,3\rangle}{\sqrt{4 + 64 + 9}} = \dfrac{\langle 2,8,3\rangle}{\sqrt{77}}$.