Math 201-13 Quize #6A Sem 041

Name: Key I.D.#__________ Serial #_____

Q1: Locate all relative extrema and saddle points, if any, of the function $f(x,y) = xy - x^2 - y^2$

$$\left.\begin{array}{l} f_x = y - 2x \\ f_y = x - 2y \end{array}\right\} \left.\begin{array}{l} f_x = 0 \\ f_y = 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} y = 2x \\ x = 2y \end{array}\right\} \longrightarrow \begin{array}{l} y = 4y \Rightarrow y = 0 \\ \text{So } x = 0 \end{array}$$

the only critical point is $(0,0)$.

$f_{xx} = -2$, $f_{yy} = -2$, $f_{xy} = 1$

$D(0,0) = (-2)(-2) - 1 = 3 > 0$ & $f_{xx} < 0$

So f has relative max. at $(0,0)$.

Q2: Find the point on the plane $2x + y - z = 2$ that is closest to the origin.

$F(x,y,z) = D^2 = x^2 + y^2 + z^2$, $g(x,y,z) = 2x + y - z - 2$

$\vec{\nabla} g = \langle 2, 1, -1 \rangle \neq \vec{0}$. & f, and g have cont. 1st par. deriv.

$\therefore \vec{\nabla} f = \lambda \vec{\nabla} g \Rightarrow \langle 2x, 2y, 2z \rangle = \lambda \langle 2, 1, -1 \rangle$

$\therefore 2x = 2\lambda$, $2y = \lambda$, $2z = -\lambda$

$x = 2y$, $z = -y$

$4y + y + y = 2 \Rightarrow y = \frac{1}{3} \Rightarrow x = \frac{2}{3}$, $x = \frac{-1}{3}$

$\therefore$ the point is $\left(\frac{2}{3}, \frac{1}{3}, \frac{-1}{3}\right)$

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Q1: Locate all relative extrema and saddle points, if any, of the function $f(x,y) = 2xy + x^2 - y^2$

$f_x = 2y + 2x$, $f_y = 2x - 2y$ $\Big\}$ $f_x = 0$, $f_y = 0$ $\Big\} \Rightarrow$

$$\begin{aligned} 2y + 2x &= 0 \\ + \quad 2x - 2y &= 0 \\ \hline 4x &= 0 \Rightarrow x = 0 \end{aligned}$$

So $y = 0$

$\therefore (0,0)$ is the only critical point of f

$f_{xx} = 2$, $f_{yy} = -2$, $f_{xy} = 2$

$D(0,0) = 2(-2) - 2^2 = -8 < 0$

So f has a saddle point at $(0,0)$.

Q2: Find the point on the plane $x + 2y + 2z = 1$ that is closest to the origin.

Use Lagrange's Multiplier

The function to be optimized is $D^2 = x^2 + y^2 + z^2 = f(x,y,z)$

the constraint is: $g(x,y,z) = x + 2y + 2z - 1$

They both are polynomials so they have Cont. 1st partial derivatives. & $\vec{\nabla} g = \langle 1, 2, 2 \rangle \neq \vec{0}$

$\vec{\nabla} f = \langle 2x, 2y, 2z \rangle \Rightarrow \vec{\nabla} f = \lambda \vec{\nabla} g$

$\therefore 2x = \lambda$, $2y = 2\lambda$, $2z = 2\lambda$

$y = 2x$ & $z = 2x$ back in the constraint eq.

$x + 4x + 4x = 1 \Rightarrow x = \frac{1}{9}$, $\Rightarrow y = \frac{2}{9}$, & $z = \frac{2}{9}$

the point is $\left(\frac{1}{9}, \frac{2}{9}, \frac{2}{9}\right)$