King Fahd University of Petroleum & Minerals
Math-101, Sec. 5 & 10
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Final Exam	**Semester 051**	**Duration: 3 hours**
# of Questions: 8	**# of Pages: 8**	
Name:	**ID #:**	**Section #: Sec. 5, 8-9 am** **Sec. 10, 9-10 am**

Q1. (10 Points - Suggested time: 20 minutes) State if each of the following statements is true or false:

1. If $\lim_{x\mapsto\infty} \frac{f(x)}{g(x)}$ exists, then $\lim_{x\mapsto\infty} f(x)$ exists.
2. If $f(x)\cdot g(x)$ is differentiable at $x=x_0$, then both $f(x)$ and $g(x)$ are differentiable at $x=x_0$.
3. If $g(x)=(x-x_0)f(x)$ has no vertical asymptote at $x=x_0$, then $f(x)$ has no vertical asymptote at $x=x_0$.
4. If $f(x)=x^e$, then $f'(x)=x^e$.
5. If $f(x):\mathbb{R}\to\mathbb{R}$, then there exists at least one point $x_0\in\mathbb{R}$ at which $f(x)$ is differentiable.
6. If $f(x)$ is continuous at $x=x_0$, then $f(x)$ is differentiable at $x=x_0$.
7. For every differentiable function $f(x)$ with $f(x)>0$ for all $x\in\mathbb{R}$, the function $\ln(f(x))$ is also differentiable.
8. If $\lim_{x\mapsto x_0} f(x)$ does not exist and $\lim_{x\mapsto x_0} g(x)$ does not exist, then $\lim_{x\mapsto x_0}(f(x)g(x))$ does not exist.
9. The graph of $P(x)=(x-2)^{4444}(x^2+1)$ is tangent to the x-axis and does not cross it.
10. If $f(x)$ is continuous on the open interval $(2,3)$, then $f(x)$ has neither an absolute maximum nor an absolute minimum on $(2,3)$.

Q2. (10 Points - Suggested time: 10 minutes) Showing all details, prove that

$$\frac{d}{dx}(\csc^{-1}(x))=\frac{-1}{|x|\sqrt{x^2-1}},\quad |x|>1.$$

Q3. (20 Points - Suggested time: 40 minutes) Find the following limits, if they exist (Showing all details):

1. $\lim_{x \mapsto 1} \frac{3^{(x^3-1)}-1}{x^3-1} =$

2. $\lim_{x \mapsto \infty} (1 + \frac{1}{2x})^{3x} =$

3. $\lim_{x \mapsto \infty} (x \cot(\frac{1}{x})) =$

4. $\lim_{x \mapsto -\infty} \frac{\cos(x^2)+x^5-x^2+1}{x^2-1} =$

Q4. (10 Points - Suggested time: 10 minutes) Find the equation of the tangent line to the curve of

$$\tan(xy^2) + x - 2y = \frac{\pi}{4} - y^2.$$

at the point $(\frac{\pi}{4}, 1)$. (Show all details).

Q5. (10 Points - Suggested time: 10 minutes) Write all what you know about the following:

1. **L'Hopital's Rule:**

2. **Roll's Theorem (and its geometric interpretation):**

3. **The 2nd Derivative Test:**

4. **Linearization of a (non-linear function):**

5. **Concavity of a function that is differentiable on $\mathbb{R}$.**

Q6. (10 Points - Suggested time: 15 minutes) A closed cylindrical can is to be made so that its total surface area is 2500 cm^2. Find the *radius* and the *height* of the can so that it has the largest possible volume. (Show all details).

Q7. (10 Points - Suggested time: 15 minutes) Given

$$f(x) = \begin{cases} 2(x^2+1) & -1 \leq x \leq 1; \\ x^3 + x + 2, & 1 < x \leq 3. \end{cases}$$

a) Show that $f(x)$ satisfies all conditions of the **mean value theorem** on $[-1, 3]$.

b) Find the value(s) of $c \in [-1, 3]$, if any, at which the tangent to the curve of $f(x)$ is parallel to the secant between $(-1, 4)$ and $(3, 32)$.

Q8. (20 Points) (Suggested time: 30 minutes) Consider

$$f(x) = \frac{\ln(x^3)}{x}$$

1. **Find each of the following: (All details should be included on the back).**

   (a) Domain$(f(x)) =$

   (b) Range$(f(x)) =$

   (c) x-intercept(s) (if any):

   (d) y-intercept (if any):

   (e) Symmetries (if any):

   (f) $\lim_{x \mapsto \infty} f(x) =$

   (g) $\lim_{x \mapsto -\infty} f(x) =$

   (h) Asymptote(s) (if any):

   (i) Critical Point(s) (if any):

   (j) Interval(s) on which $f(x)$ is increasing (if any):

   (k) Interval(s) on which $f(x)$ is decreasing (if any):

   (l) Relative Maxima (if any):

   (m) Relative Minima (if any):

   (n) Absolute Maximum (if any):

   (o) Absolute Minimum (if any):

   (p) Interval(s) on which the graph of $f(x)$ is concave up (if any):

   (q) Interval(s) on which the graph of $f(x)$ is concave down (if any):

   (r) Inflection Point(s) (if any):

2. **Draw the graph of $f(x)$ (Include the final graph below).**