King Fahd University of Petroleum & Minerals
Department of Mathematical Sciences

Math 101 - 14 & 18
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Final Exam	**Semester 041**	**3 Hours**
Name:	**ID #:**	**Section #:**

Q1. (10 Points - Suggested time: 15 minutes) State if each of the following statements is true or false:

1. If $f(x) : \mathbb{R} \to \mathbb{R}$ is increasing for $x < 3$ and decreasing for $x \geq 3$ then $f(3)$ is relative maximum.
2. If $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b) with $f(a) = f(b)$ then there exists at least one $c \in (a, b)$ with $f'(c) = 0$.
3. The graph of $f(x) = \ln(1 + x^2)$ has no horizontal tangents.
4. If $f'(x_0) = 0 = f''(x_0)$ then $f(x_0)$ is neither a local maximum nor a local minimum.
5. $\lim\limits_{x \mapsto 0}(1 + x)^{\frac{1}{x}} = 1$
6. $\sin^{-1}(x) + \cos^{-1}(x) = \frac{\pi}{2}$ for all $x \in [-1, 1]$
7. $\lim\limits_{x \mapsto \infty} \frac{x^{1,000,000}}{e^{0.000001x}} = 0$
8. The function $f(x) = 1 - x^2 - 3x^3$ has neither an absolute maximum nor an absolute minimum
9. If the graph of $f(x)$ has an inflection point at $x = x_0$ then $f''(x_0) = 0$
10. $f(x) = x + 2\sin(x)$ has no inverse

Q2. (10 Points - Suggested time: 15 minutes) Find the point(s) on the curve of $f(x) = x^{\frac{3}{2}}$ that is closest to the point $(2, 0)$.

Q3. (60 Points - Suggested time: 100 minutes) Circle the most correct answer in the following:

1. $\lim_{x \mapsto 0^-} (\frac{1}{x^2} + \frac{1}{x^3}) =$

   (a) 0
   (b) ∞
   (c) $-\infty$
   (d) 1
   (e) none of the above

2. Let $f(x) = \sqrt{x}$. The largest $\delta > 0$ for which

$$0 < |x - 4| < \delta \Rightarrow \left|\sqrt{x} - 2\right| < 1$$

   (a) 3
   (b) 5
   (c) 4
   (d) 1
   (e) none of the above

3. The function $f(x) = \begin{cases} x^2 \sin(\frac{1}{x}), & \text{for } x \neq 0 \\ 0, & \text{for } x = 0 \end{cases}$

   (a) differentiable for all $x \in \mathbb{R}$
   (b) continuous but not differentiable at $x_0 = 0$
   (c) not continuous and not differentiable at $x_0 = 0$
   (d) has a vertical asymptote $x = 0$
   (e) none of the above

4. $\lim_{x \mapsto \infty} (4^x + 7^x)^{\frac{1}{x}} =$

   (a) 4
   (b) 7
   (c) $\frac{4}{7}$
   (d) $\frac{7}{4}$
   (e) none of the above

5. The slope of the normal to the curve described by the equation $xy^3 - \frac{1}{2}xy = 1$ at $(2, 1)$ is

   (a) 10

   (b) -10

   (c) $\frac{1}{10}$

   (d) $\frac{-1}{10}$

   (e) none of the above

6. Using the local linear approximation of $f(x) = \sqrt{x}$ at $x_0 = 4$, the approximate value of $\sqrt{3.99}$ is

   (a) 1.95

   (b) 1.995

   (c) 1.975

   (d) 1.9975

   (e) none of the above

7. The function $f(x) = \frac{x^2+x-2}{x-1}$ has

   (a) a horizontal asymptote

   (b) no vertical asymptotes

   (c) one vertical asymptote and no horizontal asymptotes

   (d) no slant asymptote

   (e) none of the above

8. The value(s) of c that satisfy the Mean Value Theorem for

$$f(x) = \begin{cases} x^2 - x + 1, & \text{for} \quad 0 \leq x \leq 2 \\ 1 - x, & \text{for} \quad -3 \leq x < 0 \end{cases}$$

   (a) $c = -1$

   (b) $c = 0.4$

   (c) $c = 1$

   (d) $c = -0.4$

   (e) none of the above

9. $\lim_{x \mapsto -\infty} (1 + \frac{2}{x})^x =$

   (a) 2
   (b) 1
   (c) -1
   (d) e^2
   (e) none of the above

10. The function $f(x) = \frac{x-1}{x^2-1}$ has

   (a) irremovable discontinuity at $x_0 = 1$
   (b) removable discontinuity at $x_0 = 1$
   (c) a vertical tangent at $x_0 = 1$
   (d) a cusp at $x_0 = 1$
   (e) none of the above

11. The set S of all value(s) of k that makes the following function continuous at $x_0 = 0$

$$f(x) = \begin{cases} \frac{\tan(kx)}{x}, & \text{for } x < 0 \\ 3x + k^3, & \text{for } x \geq 0 \end{cases}$$

   (a) $S = \{0, 1, -1\}$
   (b) $S = \{-1, 1\}$
   (c) $S = \{1\}$
   (d) $S = \emptyset$ (the empty set)
   (e) none of the above

12. A particle moving on a coordinate axis with position function

$$s(t) = \frac{t^2 + 1}{t + 1} \text{ for } t \geq 0.$$

   (a) The particle is speeding up for all $t > 0$.
   (b) The particle is slowing down for all $t > 0$.
   (c) The particle speeds up for $t \in (0, \sqrt{2} - 1)$ and slows down for $t \in (\sqrt{2} - 1, \infty)$.
   (d) The particle slows down for $t \in (0, \sqrt{2} - 1)$ and speeds up for $t \in (\sqrt{2} - 1, \infty)$.
   (e) none of the above

13. If $F(x) = f(g(x^2))$ and $g(4) = 2$, $f'(2) = 3$ and $g'(4) = 5$ then $F'(2) =$

    (a) 60
    (b) 15
    (c) 30
    (d) 120
    (e) none of the above

14. Using Newton's Method to solve $x^3 - x + 1 = 0$ with $x_0 = -1$ we find out that $x_2 =$

    (a) $\frac{-31}{23}$
    (b) $\frac{-3}{2}$
    (c) $\frac{-7}{46}$
    (d) $\frac{-7}{8}$
    (e) none of the above

15. $\lim_{x \mapsto \infty} \frac{\tan^{-1} x - \frac{\pi}{4}}{x - \frac{\pi}{4}} =$

    (a) ∞
    (b) $-\infty$
    (c) 0
    (d) Does Not Exist
    (e) none of the above

16. The function $f(x) = \sqrt{x} - \sqrt{x^3}$ has:

    (a) exactly one critical point that is not stationary
    (b) one stationary point and no other critical points
    (c) exactly two critical points
    (d) more that two critical points
    (e) none of the above

17. The volume of the right circular cylinder of greatest volume that can be inscribed in a right circular cone is

    (a) $\frac{2}{3}$ volume of the cone
    (b) $\frac{1}{3}$ volume of the cone
    (c) $\frac{1}{2}$ volume of the cone
    (d) $\frac{4}{9}$ volume of the cone
    (e) none of the above

18. Assume you have a cone with radius equal to $\frac{1}{3}$ its altitude. The percentage error allowed in measuring the radius of the cone, so that the percentage error in calculating the volume will not exceed $\pm$ 0.06 % is

    (a) $\pm$ 0.03 %
    (b) $\pm$ 0.02 %
    (c) $\pm$ 0.01 %
    (d) $\pm$ 0.06 %
    (e) none of the above

19. The graph of the function $f(x) = \frac{x^3-x^2-8}{x-1}$ is concave up on

    (a) $(-\infty, 1) \cup (3, \infty)$
    (b) $(-\infty, -2) \cup (1, \infty)$
    (c) $(1, 3)$
    (d) $(1, \infty)$
    (e) none of the above

20. The function $f(x) = e^x - x$ has

    (a) no absolute maximum and no absolute minimum
    (b) an absolute maximum and an absolute minimum
    (c) an absolute maximum but no absolute minimum
    (d) an absolute minimum but no absolute maximum
    (e) none of the above

Q4. (20 Points - Suggested time: 30 minutes) Graph

$$f(x) = x\ln(x^2) \text{ for } x \in [-3, 4)\backslash\{0\}.$$

1. Find (showing all details):

   (a) $\lim_{x \mapsto 0} f(x) =$

   (b) symmetries (if any):

   (c) interval(s) on which $f(x)$ is increasing:

   (d) interval(s) on which $f(x)$ is decreasing:

   (e) relative maximum(s) (if any):

   (f) relative minimum(s) (if any):

   (g) absolute maximum (if any):

   (h) absolute minimum (if any):

   (i) interval(s) on which the graph of $f(x)$ is concave up:

   (j) interval(s) on which the graph of $f(x)$ is concave down:

   (k) the x-intercept(s) (if any):

   (l) inflection point(s) (if any):

   (m) Range($f(x)$) :

2. Draw the graph of $f(x)$ (**final** graph should be included on **this page**):