King Fahd University of Petroleum & Minerals

Math 101 - 14 & 18

Dr. Jawad Y. Abuihlail

2^{nd} Major Exam **Semester 041** **Time: 90 min.**
Name: **ID #:** **Section #:**

Q1. (10 Points - Suggested time: 5 minutes) State if each of the following statements is true or false:

1. Every monotone function has an inverse.
2. If $f(x)$ and $g(x)$ are differentiable on $(-\infty, \infty)$ then $(f \circ g)(x)$ is differentiable on $(-\infty, \infty)$.
3. If $f'(4) = 2$, $g(2) = 4$ and $g'(2) = 3$ then $(f \circ g)'(2) = 5$.
4. The inverse of a differentiable 1-1 functions $f : (-\infty, \infty) \to (-\infty, \infty)$ is differentiable everywhere.
5. If f is 1-1, $f'(2) = 4$ and $f(2) = 4$ then $\frac{df^{-1}}{dx}_{|x=4} = \frac{1}{4}$.

Q2. (10 Points - Suggested time: 10 minutes) Find the percentage error is calculating the volume of a cone with constant altitude if the radius is measured with a percentage error of $\pm$ 5 %.

Q3. (20 Points - Suggested time: 20 minutes)

1. Find the point(s) in the first quadrant at which the tangent line to the curve $x^3 - xy + y^3 = 0$ is parallel to the x-axis.

2. Find $(\frac{d^2y}{dx^2})_{|_{(1,1)}}$ if

$$2x^2y - y = 1.$$

Q4. (20 Points - Suggested time: 20 minutes) Use the **definition** to

1. find $f'(x)$ for $f(x) = 2x^3 - x + 1$.

2. show that $(\cos x)' = -\sin x$.

Q5. (20 Points - Suggested time: 15 minutes) Assume we have a cylinder with height equal to three times its radius. If the area of the cylinder is increasing at a rate of $5\pi \ ft^2/\sec$ when $r = 2$, how fast is the altitude of the cylinder increasing at that instance.

Q6. (20 Points - Suggested time: 20 minutes) Determine what makes each of the following functions non-differentiable at $x_0 = 0$ (**Hint:** you may deduce the answer from the graph of each function):

1. $f(x) = \begin{cases} 2x - 1, & x \geq 0 \\ -(1 + 3x), & x < 0 \end{cases}$

2. $f(x) = \begin{cases} x^2 + 1, & x \geq 0 \\ x - 2, & x < 0 \end{cases}$

3. $f(x) = \sqrt[5]{x}$

4. $f(x) = \sqrt[5]{x^2}$

GOOD LUCK