King Fahd University of Petroleum & Minerals

Math 101 - 14 & 18

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First Major Exam **Semester 041**

Name: **ID #:** **Section #:**

Q1. (10 Points - Suggested time: 5 minutes) State if each of the following statements is true or false:

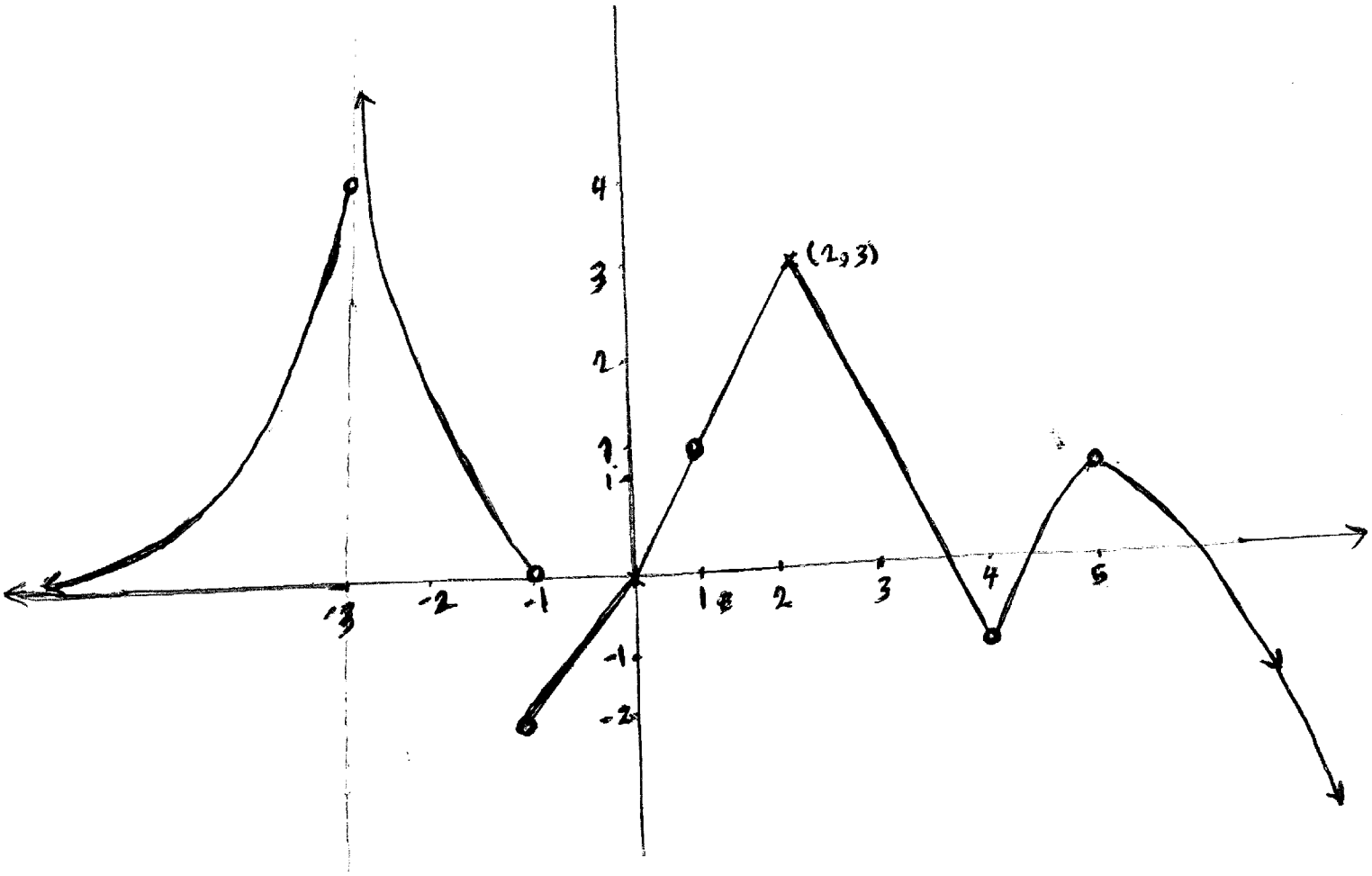
1. If $\lim_{x\mapsto\infty} f(t) = 2$, then $\lim_{x\mapsto 0} f(\frac{1}{t}) = 2$.
2. If $f(x)$ and $g(x)$ are continuous at $x = a$ then $\frac{f(x)}{g(x)}$ is continuous at $x = a$.
3. The composition of any two continuous functions is continuous.
4. There exists a function $f(x) : [0, 2\pi] \to \mathbb{R}$, whose graph have infinite number of vertical asymptotes.
5. If $\lim_{x\mapsto a} f(x) = 0$ and $\lim_{x\mapsto a} g(x) = 0$ then $\lim_{x\mapsto a} \frac{f(x)}{g(x)}$ does not exists.

Q2. (10 Points - Suggested time: 10 minutes) Sketch one possible graph for a function $f(x)$ satisfying the following properties:

1. Domian(f) = Range$(f) = \mathbb{R}$.
2. $f(0) = 0$ and $\lim_{x\mapsto 0} f(x) = 3$.
3. $\lim_{x\mapsto\infty} f(x) = -\infty$.
4. The graph of $f(x)$ has the x-axis as a horizontal asymptote.
5. $\lim_{x\mapsto 4^-} f(x) = -1$.
6. The graph of $f(x)$ has the line $x = 4$ as a vertical asymptote.

Q3. (20 Points - Suggested time: 15 minutes) Depending on the graph of the function $y = f(x)$ below, find the following (if they exist):

1. Doamin$(f(x))$:
2. Range$(f(x))$:
3. $\lim_{x \mapsto 1} f(x)$:
4. $\lim_{x \mapsto 2^-} f(x)$:
5. $\lim_{x \mapsto 4} (f(x))^2$:
6. $\lim_{x \mapsto -1^+} f(x)$:
7. $\lim_{x \mapsto -3} \sqrt{f(x)}$:
8. $\lim_{x \mapsto 5} 2f(x)$:
9. The horizontal asymptotes:
10. The vertical asymptotes:



Q4. (40 Points - Suggested time: 20 minutes) Find the following limits, if they exist (Showing all details):

1. $\lim_{x \mapsto \infty} \frac{2x^{99}-x^{39}+2}{-3x^{88}+x^3-1} =$

2. $\lim_{x \mapsto -\infty} \frac{\sqrt[3]{x^3-x}}{2x} =$

3. $\lim_{x \mapsto 2^-} \frac{x^2-x-2}{x-2} =$

4. $\lim_{x \mapsto 0} \frac{\sin 3x}{x^2} =$

Q5. (10 Points - Suggested time: 10 minutes) Show (using the definition) that

$$\lim_{x \mapsto 1}(x^2 - 3) = -2$$

Q6. (10 Points - Suggested time: 10 minutes) Draw the function

$$f(x) = 2\tan(x - \frac{\pi}{4})$$

on the interval $[-2\pi, 2\pi]$ indicating its range, x-intercepts, y-intercepts and vertical asymptotes (in the given interval).