King Fahd University of Petroleum & Minerals

Math 202 - 4 & 5

Dr. Jawad Y. Abuihlail

Final Exam **Semester 042** **3 Hours**

Name: **ID #:**

Section #: 4 9:00 - 10:00 *or* **5 10:00 - 11:00**

Q1. (15 Points - Suggested time: 15 minutes) State if each of the following statements is true or false:

1. If $h(x) := f(x) + g(x)$ is analytic at x_0, then $f(x)$ and $g(x)$ are also analytic at x_0.
2. Every separable ODE has a solution.
3. Every eigenvalue has an infinite number of eigenvectors.
4. If the Wronskian of a set of solutions of a given higher order ODE is zero at $x_0 \in I$ (where I is some open interval included in the interval of solution), then the set is not fundamental on I.
5. Every 2nd order ODE with polynomial coefficients has a solution.
6. Every Cauchy Euler equation can be transformed into one with constant coefficients.
7. If $y_1 = e^x$ and $y_2 = xe^x$ are solutions of a homogenous 2nd order ODE with constant coefficients, then $m = 1$ is a root with multiplicity 2 of the associated auxiliary equation.
8. If y_p is a particular solution of a given ODE, then $2y_p$ is also a solution of that equation.
9. An eigenvalue λ with multiplicity 3 can have at most 3 linearly independent eigenvectors.
10. If a matrix A has the property $A^3 = 0$, then $e^A = I + A + \frac{1}{2}A^2$.

Q2. (10 Points - Suggested time: 15 minutes) Solve the following IVP

$$y' - x^2 y = x^2;\ y(0) = 0$$

Q3. (40 Points - Suggested time: 80 minutes) Find the solution of the following ODE's

1.
$$y' = \frac{x - 2x^2 - y}{x - y^2 + y}$$

2.

$$y'' - 3y' + 2y = e^x - e^{2x} + x$$

3.

$$2x^2y'' - xy' + 2y = 0$$

4.

$$xy'' + y' - y = 0.$$

Q4. (20 Points - Suggested time: 30 minutes) Solve the following system of ODE's

$$\begin{aligned} \frac{dx}{dt} &= x - y + 3t \\ \frac{dy}{dt} &= 2x + y - 1 \end{aligned}$$

Q5. (15 Points - Suggested time: 30 minutes) The radioactive isotope of lead, $\mathbf{Pb-290}$, decays at a rate proportional to the amount of present at time t and has a half-life of 3.3 hours. If 1 gm of this isotope is present initially, how long will it take for 90% of the lead to decay?!!