

King Fahd University of Petroleum & Minerals
Department of Mathematical Sciences

MATH-533: Complex Variables I
Spring Semester 2004 (032)

Dr. Jawad Abuhlail

Homework 2

Due 29.3.2004

Q1. Let $Q(z)$ be a polynomial with n distinct roots $\alpha_1, \dots, \alpha_n$ and $P(z)$ be a polynomial of degree $< n$. Show that

$$\frac{P(z)}{Q(z)} = \sum_{k=1}^n \frac{P(\alpha_k)}{Q'(\alpha_k)(z - \alpha_k)}.$$

If $c_1, \dots, c_n$ are given complex numbers, use the formula above to show that there exists a unique polynomial P of degree $< n$ with $P(\alpha_k) = c_k$ for $k = 1, \dots, n$.

Q2. If $\lim_{n \rightarrow \infty} z_n = A$, prove that

$$\lim_{n \rightarrow \infty} \frac{1}{n} (z_1 + \dots + z_n) = A.$$

Q3. Discuss completely the convergence and uniform convergence of $\{nz^n\}_{n=1}^{\infty}$.

Q4. Let R be the radius of convergence for $\sum a_n z^n$. What is the radius of convergence for $\sum a_n z^{2n}$ and $\sum a_n^2 z^n$?

Q5. If $f(z) = \sum a_n z^n$, what is $\sum n^3 a_n z^n$?

Q6. Find the values of z , for which the following series converges

$$\sum_{n=0}^{\infty} \frac{z^n}{1 + z^{2n}}.$$

Q7. Find the radius of convergence of the following power series: $\sum \frac{z^n}{n!}$, $\sum n! z^n$, $\sum z^{n!}$.

GOOD LUCK