King Fahd University of Petroleum & Minerals
Department of Mathematical Sciences

Math 101 - 2 & 7

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Third Major Exam **Semester 031**
Time: 17:20-18:50 pm, Wednesday 31.12.2003

Name: **ID #:** **Section #:**

Q1. (10 Points) (Suggested time: 10 minutes) State if each of the following statements is TRUE or FALSE:

1. There may exist two inverses of a 1-1 function.
2. $f(x) = x + x^5$ has an inverse.
3. $\frac{d}{dx}(\cot^{-1}(x)) = \frac{1}{1+x^2}$ for all $x \in \mathbb{R}$.
4. If $f(x) : (a, b) \longrightarrow R$ is 1-1 and differentiable, the $g := f^{-1}$ is differentiable on R.
5. If $f : D \longrightarrow R$ and $g : R \longrightarrow S$ have inverses, then $g \circ f : D \longrightarrow S$ also has an inverse.

In solving each of the following problems, show all details of the solution:

Q2. (40 Points) (Suggested time: 30 minutes) Solve each of the following questions

1. Let $f(x) = 1 - x^3$.
   (a) Show that f has inverse.
   (b) Find $g := f^{-1}$ (indicating its domain and range).
   (c) Find $\frac{d}{dx}(f^{-1}(x))$.

2. Let f be a 1-1 function, $g := f^{-1}$, $f(1) = 2$, $f'(1) = 3$, $f(2) = 4$ and $f'(2) = 5$. Assuming that

$$F(x) = (g(x))^2 - \frac{1}{f(x)}.$$

find $F'(2)$.

3. Solve the equation

$$\frac{e^x + e^{-x}}{2} = 3.$$

4. Consider the curve C given by the equation:

$$x \tan^{-1} y = 1.$$

Find the equation of the tangent line to the curve at the point $(\frac{4}{\pi}, 1)$.

Q3. (**20 Points**) (**Suggested time: 10 minutes**)
Evaluate the following limits

1.

$$\lim_{x \mapsto 0^+} (1 + x^2)^{\frac{1}{x}}.$$

2.

$$\lim_{x \mapsto \infty} \frac{x^2 - x}{\ln(\ln x)}.$$

Q4. (10 Points) (Suggested time: 5 minutes)

Find the value of k, such that $f(x) = \sqrt{x} - k$ and $g(x) = \ln x$ share the same tangent at their point of intersection.

Q5. (**10 Points**) **(Suggested time: 10 minutes)** Graph

$$g(x) = e^{(2-x)} - 4.$$

indicating the domain, the range, the x-intercept, the y-intercept and the asymptotes (if any).

Q6. (10 Points) (Suggested time: 5 minutes)
Show that

$$\frac{d}{dx}\cos^{-1}(x) = \frac{-1}{\sqrt{1-x^2}} \text{ for all } x \in (-1,1).$$

Bonus (10 Grades) Let $y = \tan^{-1} x$. Show that

$$y'' = -2\sin y \cos^3 y.$$

GOOD LUCK